MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor A. S. EDDINGTON, F.R.S., PRESIDENT, in the Chair.

Ronald Edward Beare, Nautical Instructor, 16 Newcombe Road, Southampton;

Rev. William Edwards, The Manse, Caerau, near Bridgend, Glamorganshire;

Rev. John Hixon Irving, Cressington Avenue, Higher Tranmere, Cheshire;

Richard Marshall, F.R.Met.Soc., 118 Ribblesdale Road, Streatham, London, S.W. 16;

Mrs. Howard Pease, Otterburn Tower, Otterburn, Northumberland; and

Rev. Henry John Wilkins, M.A., D.D., Westbury on Trym, Bristol,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Harold D. Babcock, Mount Wilson Observatory, Pasadena, California (proposed by G. E. Hale);

Bernard Benfield, Ph.B., 719 Rialto Building, San Francisco (proposed by W. W. Campbell);

Captain Percy Francis Ward Blake, M.B.E., Beechworth, The Green, Wanstead, London, E. 11 (proposed by Stanley Fielder);

Clifford Charles Crump, B.S., M.A., Ph.D., Professor of Astronomy and Director of the Perkins Observatory, Ohio Wesleyan University, Delaware, Ohio (proposed by E. E. Barnard);

Charles S. Cushing, 821 First National Bank Building, San Francisco (proposed by W. W. Campbell);

Harcourt Leslie Dilks, Spring Bank, Broad Oak Park, Worsley, Manchester (proposed by W. Porthouse);

William Snyder Eichelberger, Ph.D., Director of the Nautical Almanac, Washington, D.C. (proposed by F. Schlesinger);

Ferdinand Ellerman, Mount Wilson Observatory, Pasadena, California (proposed by G. E. Hale);

John Stratford Fleetcroft, 44 Middleton Square, London, E.C. (proposed by A. G. Fellows);

Miss Caroline Furness, Ph.D., Professor of Astronomy, Vassar College, Poughkeepsie, New York (proposed by F. Schlesinger);

Thomas Gray, Ashmead, Stockton Lane, York (proposed by E. H. Grove-Hills);

Miss Margaret Harwood, Director of the Maria Mitchell Observatory, Nantucket, Massachusetts (proposed by Harlow Shapley);

John H. Hindle, Union Engineering Works, Haslingden, Lancs. (proposed by W. Porthouse);

James F. Hough, M.A., The School House, Brentwood (proposed by Joseph Hough);

John Richard Hudson, B.A., B.A.I., A.M.I.C.E.I., St. Ronan's, Clondalkin, Co. Dublin (proposed by J. T. Jackson);

Francis P. Leavenworth, M.A., Professor of Astronomy in the University of Minnesota, Minneapolis, U.S.A. (proposed by E. E. Barnard);

Frank Bowers Littell, Sc.D, Professor of Mathematics, U.S. Naval Observatory, Washington, D.C. (proposed by F. Schlesinger);

Miss Frances Lowater, B.S., Ph.D., Assistant Professor of Physics, Wellesley College, Wellesley, Mass. (proposed by E. E. Barnard);

Dr. Adriaan van Maanen, Mount Wilson Observatory, Pasadena, California (proposed by C. E. St. John);

Richard J. McDiarmid, Ph.D., Dominion Observatory, Ottawa, Canada (proposed by Otto Klotz);

Dr. Paul W. Merrill, Mount Wilson Observatory, Pasadena, California (proposed by C. E. St. John);

Willis Isbister Milham, Ph.D., Field Memorial Professor of Astronomy in Williams College, Williamstown, Mass. (proposed by J. Tatlock);

John A. Miller, Ph.D., Professor of Astronomy, Swarthmore College, Penn., and Director of the Sproul Observatory (proposed by F. Schlesinger);

Daniel Walter Morehouse, B.S., M.S., Ph.D., Professor of Physics and Astronomy, Drake University, Des Moines, Iowa (proposed by E. E. Barnard);

Jason John Nassau, Ph.D., Assistant Professor of Mathematics and Astronomy, Case School of Applied Science, Cleveland, Ohio (proposed by C. S. Howe);

Francis G. Pease, Mount Wilson Observatory, Pasadena, California (proposed by G. E. Hale);

Earl C. Slipher, Lowell Observatory, Flagstaff, Arizona (proposed by G. H. Hamilton);

Charles Campbell Smith, M.A., Dominion Observatory, Ottawa, Canada (proposed by Otto Klotz);

Thomas J. Smith, c/o John Browning, 37 Southampton Street, Strand, London, W.C. 2 (proposed by E. H. Grove-Hills);

Frank Dickinson Urie, B.A., Assistant Director, Elgin Observatory, Elgin, Illinois (proposed by E. E. Barnard);

C. I. Veale, Thomason College, Roorkee, India (proposed by W. H. Connell);

R. Stretton Webb, The Anchorage, Market Square, Paarl, S. Africa (proposed by G. T. McCaw);

James Simms Wilson, c/o Troughton & Simms, Charlton, London, S.E. 7 (proposed by E. H. Grove-Hills); and

Miss Anne Sewell Young, B.L., M.S., Ph.D., Professor of Astronomy and Director of Williston Observatory, Mount Holyoke College, South Hadley, Mass. (proposed by E. E. Barnard).

Sixty-five presents were announced as having been received since the last Meeting, including, amongst others :—

Tycho Brahe, Opera Omnia, tomus 4, pt. 2 ; tomus 5 (presented by Dr. Dreyer).

Pomona College Astronomical Society, Publication vol. 7, No. 1.

Tacubaya Observatory, Astrographic Chart (10 charts).

The President read the following Resolution, which had been passed by the Council at their Meeting that afternoon, relating to Sir Charles Parsons' gift of £2500 to the Society, and invited the Meeting to associate itself with this Resolution, which was done :

"Resolved that the thanks of the Council be given to the Hon. Sir Charles Parsons, K.C.B., F.R.S., for his generous gift to the Society of £2500 5 per cent. War Stock (1929–47).

On the Relationship between Solar Prominences and the Corona.
By Major William J. S. Lockyer, M.A., Ph.D.

In 1903 June I communicated a paper* to the Society on "A Probable Relationship between the Solar Prominences and Corona," in which I attempted to point out that the various forms of the corona, as seen during total eclipses of the sun, are closely associated with the positions, as regards latitude, of the chief zones of the solar prominences.

In another paper† entitled "Prominence and Coronal Structure" I brought together some data to show that in the lower reaches of the sun's atmosphere there was also an intimate association between prominences and the neighbouring coronal matter.

It is proposed in this paper to bring the material, in both cases, up to date as far as possible, to see whether the new data endorse the earlier conclusions.

The first research was based on the percentage frequency of prominences as determined from observations made by Secchi and Tacchini at Rome, and Ricco and Mascari at Catania and Palermo. The forms of coronas discussed extended over the period 1858 to 1901, and included observations made at twenty-three total eclipses.

* *Monthly Notices R.A.S.*, 63, 481–488.
† *Proc. Roy. Soc.*, A, 80, 178 (1908).

Since the last-mentioned date the solar cycle has passed through two epochs of maximum and one minimum, and nine total solar eclipses have been observed. Further, quite a new series of prominence observations has been systematically made by Mr. John Evershed, first at Kenley from 1890 to 1904, and afterwards at the Solar Physics Observatory at Kodaikánal, S. India, from 1905 up to date.

The Prominence Data.

The prominence data here discussed are taken from two different sources. Those from 1890 to 1914 have been published in one of the *Memoirs of the Kodaikánal Observatory*,* and the remainder have been extracted from the *Kodaikánal Observatory's Bulletins*.

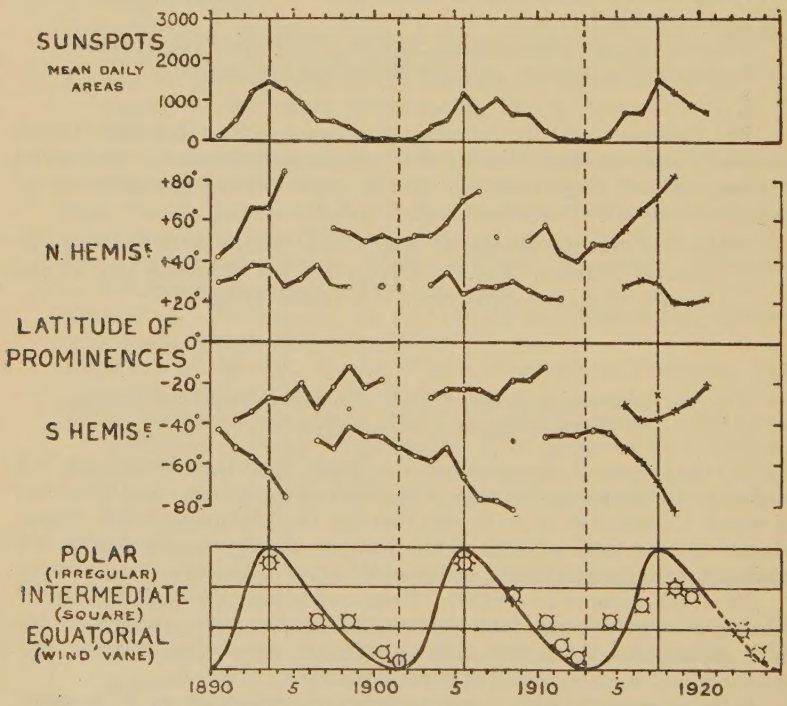


FIG. 1.—Comparison between Prominence Zones and Forms of the Corona.

The movements in latitude of the four zones of prominences from 1890 to 1914 are illustrated in the *Memoir* in the form of a diagram (diagram vii.), and these zones are reproduced here (fig. 1) by thick lines connecting small circles. The positions of the zones from 1914 onwards have been determined for each year from the curves in the

* Vol. i. part ii., "Results of Prominence Observations," by John Evershed and Mary Acworth Evershed (1917).

Bulletins (published half-yearly) illustrating the distribution of prominences, both observed and photographed, for each zone of 5° of latitude. In fig. 1 these zones are indicated by crosses with connecting thick lines.

It is interesting to note that this new series of observations completely endorses the results obtained from the Italian observations, as the following statement from Mr. Evershed's *Memoir* (p. 63) indicates:—

"The progress of the high-latitude zones is very clear, the poleward march at sunspot maximum, the subsequent disappearance, reappearance, and persistence without much change of latitude until next maximum, when they again rush to the poles, disappear and reappear: but the trend of the low-latitude zones is not quite so clear, for there are greater irregularities in the latitude variation, and sometimes these zones are multiple. On the whole, however, these zones, which always remain within sunspot regions, seem to approach one another, and die out near the equator at minimum, as sunspots do. A few sporadic zones, which do not appear to play a part in the history of these main zones, have been inserted as dots on the diagram but not connected by lines with the others.

"These results are in agreement with the law of prominence distribution discovered by Professor Ricco in 1891,* and confirmed by Sir Norman Lockyer and Dr. W. Lockyer in a paper read before the Royal Society in March 1903."†

While the above remarks apply only to the curves up to the year 1914, it will be seen that they are equally applicable to those which have been extended from 1914 to the year 1920.

The Corona Data.

The forms of the corona are here, as before, divided into three main types. The "Polar" includes those irregular forms in which streamers may be seen all round the solar disc: the "Equatorial," where the streamers are restricted to low solar latitudes and the poles are conspicuous for the large spread in latitude of the polar rifts, a form commonly termed "wind-vane": and the "Intermediate," where the streamers are chiefly half-way between the poles and the equator, giving the appearance of a "square" corona (fig. 2).

In stating the general form of the corona, either from eye observations or photographs, it must not be forgotten that one is not looking at a *section* of the solar atmosphere, but at a portion of a sphere. Thus it may and probably does happen that when polar rifts do exist, they may be partially or totally obliterated from sight by a long streamer or streamers emanating from mid-latitudes on that side of the sun nearest the earth, as will be seen from the "intermediate" form shown in fig. 2. This streamer being in the line of sight, and extending probably to a great distance radially from the solar surface, would there-

* *Memorie della Società degli Spettroscopisti Italiani*, 20, 135.

† *Proc. Roy. Soc.*, 71, 446, and *Monthly Notices R.A.S.*, 63, Appendix No. 2. See also Ricco, *Mem. della Società degli Spettros. Italiani*, 32, and Mascari, *ibid.*, 33.

fore hide the rifts. The conclusion might be drawn that as the rifts were not visible the form of the corona approximated more to the "polar" than to the "intermediate" type.

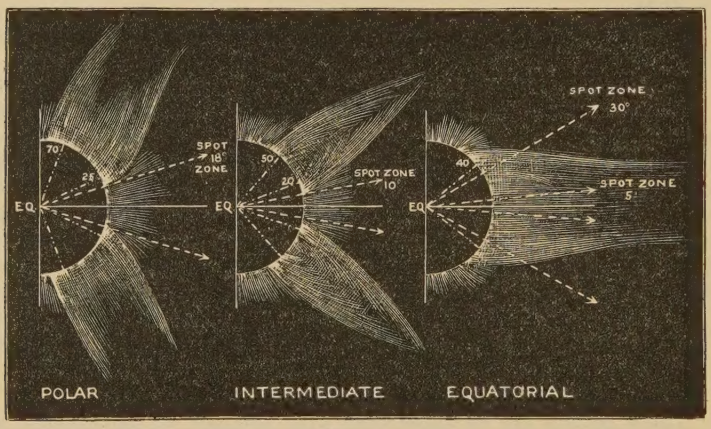


FIG. 2.

The absence of polar rifts may be due to two causes:—

(1) Prominences at the poles producing streamers.

This would occur in the true "polar" forms as seen in fig. 2.

(2) Streamers in the line of sight hiding the rifts.

This condition might often occur in "intermediate" forms, but seldom in "equatorial" forms (see fig. 2).

The following table contains a list of all the coronas observed since 1892, with their dates of occurrence. The third column gives the authority and reference to the form of the corona as described in the last column.

Forms of Corona 1893 *to* 1919.

Year.	Month.	Reference and Authority.	Form of Corona.
1893	Apr. 15	Monthly Notices R.A.S., vol. xvi. p. 201. A. Everett.	Maximum.
,,	,,	Astr. and Astroph., vol. xii. p. 645. Deslandres.	Characteristic of max. sunspot periods.
,,	,,	Astr. and Astroph., vol. xii. p. 462. Pickering.	Confirmation of 1871 (maximum).
1896	Aug. 9	Monthly Notices R.A.S., vol. lvii. p. 42. W. H. Wesley.	Resemblance to that of 1886 (intermediate).
1898	Jan. 22	Phil. Trans. R.S., vol. excvii., A. Sir N. Lockyer.	Intermediate.
,,	,,	Memoir of the Kodaikánal Obsy., vol. i. part ii. p. 65. J. Evershed.	Intermediate or square.
1900	May 28	Phil. Trans. R.S., vol. excviii., A, p 413. Sir N. Lockyer.	Minimum. Wind-vane =1878.
,,	,,	Memoir of the Kodaikánal Obsy., vol. i. part ii. p. 66. J. Evershed.	Equatorial or wind-vane.
1901	May 18	Monthly Notices R.A.S., vol. lxii., App. p. 22. H. F. Newall.	Minimum.

Forms of Corona—continued.

Year.	Month.	Reference and Authority.	Form of Corona
1901	May 18	Proc. Roy. Soc., vol. lxix. p. 244. F. W. Dyson.	"The polar plumes and the coronal streamers are well brought out" (in the photographs).
1905	Aug. 30	Report of the S.P.O. Expedition to Palma Majorca. Sir N. Lockyer.	Maximum.
,,	,,	Memoir of the Kodaikánal Obsy., vol. i. part ii. p. 66. J. Evershed.	Maximum.
,,	,,	British Astr. Ass. Total Solar Eclipse Expedition of 1905. A. L. Cortie.	Maximum.
1908	Jan. 3	Report of the Solar Eclipse Expedition by F. K. McClean, p. 11. W. H. Wesley.	Nearly of the maximum type.
,,	,,	Lick Observatory Bulletin No. 131. W. W. Campbell.	Much nearer form accompanying sunspot maxima than minima.
1910	May 8	Monthly Notices R.A.S., vol. lxx. p. 650. W. H. Wesley.	Intermediate.
1911	Apr. 29	Report of the Solar Eclipse Expedition to Vavau, p. 24. W. J. S. Lockyer.	Minimum. Wind-vane.
,,	,,	The Observatory, vol. xxxiv. p. 245. A. L. Cortie.	Characteristic of the sunspot minimum.
1912	Apr. 17	Nature, vol. lxxxix. p. 193. Worthington.	Wind-vane.
1914	Aug. 21	Monthly Notices R.A.S., vol. lxxv. p. 316. A. Fowler.	Intermediate.
,,	,,	Monthly Notices R.A.S., vol. lxxv. p. 131. Jones and Davidson.	Resemblances to 1898 (intermediate) and to 1901 (equatorial).
,,	,,	Monthly Notices R.A.S., vol. lxxv. p. 116. A. L. Cortie.	Corresponds neither to a "maximum" nor a "minimum" corona.
1916	Feb. 3	Monthly Notices R.A.S., vol. lxxvii. p. 67. C. D. Perrine.	Intermediate (=1898 intermediate).
1918	June 8	Monthly Notices R.A.S., vol. lxxix. p. 276. A. Fowler.	Intermediate.
,,	,,	Journ. of the Roy. Ast. Soc. of Canada, vol. xii. p. 346. C. A. Chant.	Not quite maximum.
1919	May 29	Monthly Notices R.A.S., vol. lxxx. p. 12. A. C. D. Crommelin and C. R. Davidson.	Blend of maximum and minimum.
,,	,,	The Observatory, vol. xlii. p. 371. A. C. D. Crommelin.	Intermediate.
,,	,,	Astro-phys. Jour., vol. li. p. 1. C. G. Abbot and A. F. Moore.	Intermediate.

Comparison of Prominence Zones with Forms of Corona.

The lower portion of fig. 1 illustrates the positions of all the above forms of the corona in relation to a diagrammatic sunspot curve made after the curve showing the mean daily area of sunspots depicted at the top of the diagram. The data for this latter curve have been extracted from the *Monthly Notices of the Royal Astronomical Society*, in which the Greenwich sunspot results have been published. The last three points

on the diagram for the years 1918, 1919 and 1920, denoted by crosses, are the approximate areas not yet finally deduced.

The vertical continuous and broken lines in the figure indicate the epochs of sunspot maximum and minimum respectively.

The forms of the corona, taken from the table above, are placed in three horizontal strips, the "polar" being at the top, "intermediate" in the middle, and "equatorial" at the bottom: they are also placed horizontally according to the years of their occurrence.

It will be seen that the forms follow the sunspot curve very regularly as this curve proceeds through the various strips. It is with the prominence curves and not the sunspot curve that the comparison is desired to be made to indicate the connection between the zones of prominence activity and the forms of the corona. In my earlier publication I pointed out that sunspots do not appear nearer the poles than latitude about $\pm 35^\circ$ (see fig. 2), and that large coronal streamers and prominent rays are often situated in much higher latitudes than these, in fact at times very near and at the poles of the sun and, consequently, quite outside the regions of spot activity.

In making the comparison between the prominence zones and the corona forms, the first point to be noted is that the prominence zones in one or both hemispheres reach latitudes above 60° in the years 1892 to 1894, 1905 to 1908, and 1916 to 1918. It is at these epochs that the corona should exhibit a "polar" or "maximum" form.

At the first of these epochs there is only one eclipse available, namely, 1893, and this is universally described as of a maximum type (see preceding table).

At the second epoch two eclipses were observed, namely, 1905 and 1908: the first of these was undoubtedly of the "polar" type. There is also little doubt that the 1908 corona was nearly, if not quite, of this type also, Mr. Wesley stating that it was "more nearly of the maximum type than any with which it can be compared."

This eclipse is of special interest here, because, although Mr. Evershed* concedes that the coronas of 1898, 1900, 1901, and 1905 "confirm Dr. Lockyer's theory that there is a connection between prominence latitudes and coronal types," he considers that that of 1908 does not.

Mr. Evershed's contention is that while his prominence curves show the existence of polar prominences in the high southern latitudes which would account for the streamers there, there is no similar record of prominences in very high northern latitudes (see fig. 1) to explain the presence of the streamers observed there.

Referring to his prominence curves for the year 1908,† which show the distribution of prominence areas for every 5° of latitude, these indicate that the prominence area in latitude 60° is nearly equal in amount in both hemispheres: towards the north pole the areas certainly diminish rapidly, disappearing altogether in latitude 80°, while they remain fairly considerable in extent towards the south pole, although always less in amount than in latitude 60°.

* *Memoirs of the Kodaikánal Observatory*, vol. i., part ii., pp. 65–67.

† *Ibid.*, diagram xi.

I would suggest that on this occasion we have a condition to which reference has been made in the earlier part of this paper.

It is not improbable that the streamers observed near the north pole may have had their bases situated in very much lower latitudes, even in latitudes 50° to 60°, where prominences were known to exist. Streamers of considerable length and projecting radially from the solar surface on that hemisphere turned towards the earth (see fig. 2) would be seen in the line of sight, and would appear projected at and above the solar pole: they would cover the polar area and partially or completely hide the polar rifts if they were existent there.

Coming to the last epoch, namely, 1916 to 1918, two eclipses are available—those in 1916 and 1918. In 1916 the higher latitude prominence zones were conspicuous features in about latitude 60° in both hemispheres. These conditions should produce an "intermediate" form verging on a "polar" form. The actual corona was described as "intermediate" in type.

In 1918 both prominence zones lie in about latitude 80° in each hemisphere; they were conspicuous during the first half of the year, but nearly non-existent during the second half. This condition should produce a "polar" type. The general description is given as "intermediate," but, as will be seen from Mr. Chant's * remarks, it was not far from the "polar" type. He writes:—

"To me the long streamers on opposite limbs seem to indicate not a typical maximum corona such as seen in 1893 and 1905. From it I would have judged that the maximum had passed by perhaps a year or more. . . ."

The photograph which he reproduces is very similar to that published by Professor W. W. Campbell,† and these exhibit a form which may be described as a mixture between "polar" and "intermediate."

Turning now to the two epochs about sunspot minimum, five eclipses are available, namely, 1900, 1901, and 1911, 1912, 1914. At these times the chief zones of prominences in both hemispheres are in about latitudes 40°–45°, while a lesser zone is dying out in about latitude 20° or has disappeared altogether. This condition should produce an "equatorial" type of corona. All the eclipses for these years are described as of the "equatorial" or "wind-vane" form, with the exception of that of 1914. There seemed evidently some difficulty in deciding the form of this corona, for Messrs. H. S. Jones and C. R. Davidson stated that it exhibited resemblances to 1898 (intermediate) and to 1901 (equatorial), and Father Cortie wrote that it corresponded neither to a "maximum" nor a "minimum" corona.

With regard to all the other "intermediate" forms, it will be seen that they all find their respective places between the successive "polar" and "equatorial" forms. In nearly all cases there are two prominent zones of prominence activity in middle and low latitudes.

The view that the various forms of the corona are closely associated with the active zones of prominences seems to be well endorsed by these new observations.

Two total eclipses will occur in the near future, namely, one this year and another in 1923. It is not without interest, therefore, to

* *Journal of the R.A.S. of Canada*, **12**, 346, 347.
† *Pub of the Astr. Soc. of the Pacific*, **30**, 230.

insert their expected forms in fig. 1. It will be seen that the corona of 1922 should assume a form of the "intermediate" type verging on the "equatorial" type, while that of 1923 should be typical of the "equatorial" type.

Prominences and the Lower Corona.

Turning attention now to the lower regions of the solar atmosphere, there seems very little or no doubt about the intimate association between prominences and coronal matter. A considerable amount of further evidence has accumulated since the year 1908.

The following brief extracts out of many indicate that numerous eclipse observers have been led to point out the close relationship between these two solar phenomena. Thus Professor W. W. Campbell,* in describing his photographs of the corona taken during the eclipse of 1918, says:—

"The corona as recorded by the 40-foot camera was remarkable for the sheaths of streamers which surround all of the principal prominences. . . . After observing them with care it seems impossible to question that the forces in the sun responsible for the prominences are the forces which are responsible for the coronal streamers situated near the prominences."

Further evidence is given by Professor C. D. Perrine, who, writing of the 1916 corona, says †:—

"Five groups of prominences are shown at the bases of the four principal wings of the corona. A series of well-marked hoods is shown over the prominences in the S.W. quadrant."

In the corona photographs of 1919 Messrs. A. C. D. Crommelin and C. R. Davidson ‡ note that

"the prominence is itself encircled by a coronal ring which nearly coincides with it at each end, but rises in the middle to about twice its height above the limb."

Lastly, Messrs. H. S. Jones and C. R. Davidson point out that the photographs they secured during the 1914 eclipse §

"show the corona in extension, and also many interesting details in the structure. Of particular interest is a coronal arch proceeding from a large prominence in the S.E. quadrant."

Results of Solar Prominence Observations, 1921. By A. M. Newbegin.

The results of prominence observations in this paper are reduced from completed observations only. In spite of the very sunny conditions of last year, there were many days when the atmosphere was so unsteady and seeing so bad, that faint prominences could not be observed sufficiently well to make reliable measurements; these observations are, therefore, not included in the records giving the statistics.

* *Pub. of the Astr. Soc. of the Pacific*, 30, 229.
† *Monthly Notices R.A.S.*, 77, 67.
‡ *Ibid.*, 80, 12. § *Ibid.*, 75, 131.

TABLE I.

1921.

MONTH	DAYS OF OBSERVATION	No. OF PROMINENCES	MEAN AVERAGE	MEAN AREA	MEAN HEIGHT
JANUARY	5	31	6·20	3·79 sq.m	38″·71
FEBRUARY	8	51	6·37	3·13 "	36·66
MARCH	7	50	7·14	4·10 "	31·90
APRIL	15	91	6·06	3·66 "	39·45
MAY	16	117	7·31	3·40 "	35·47
JUNE	16	128	8·00	2·75 "	34·72
JULY	18	165	9·16	3·66 "	35·30
AUGUST	7	44	6·28	2·97 "	34·88
SEPTEMBER	9	82	9·11	4·29 "	36·95
OCTOBER	8	83	10·37	4·59 "	35·54
NOVEMBER	5	47	9·40	3·41 "	36·08
DECEMBER	6	45	7·50	4·19 "	39·33
YEAR	120	934	7·74	3·66 "	36·25

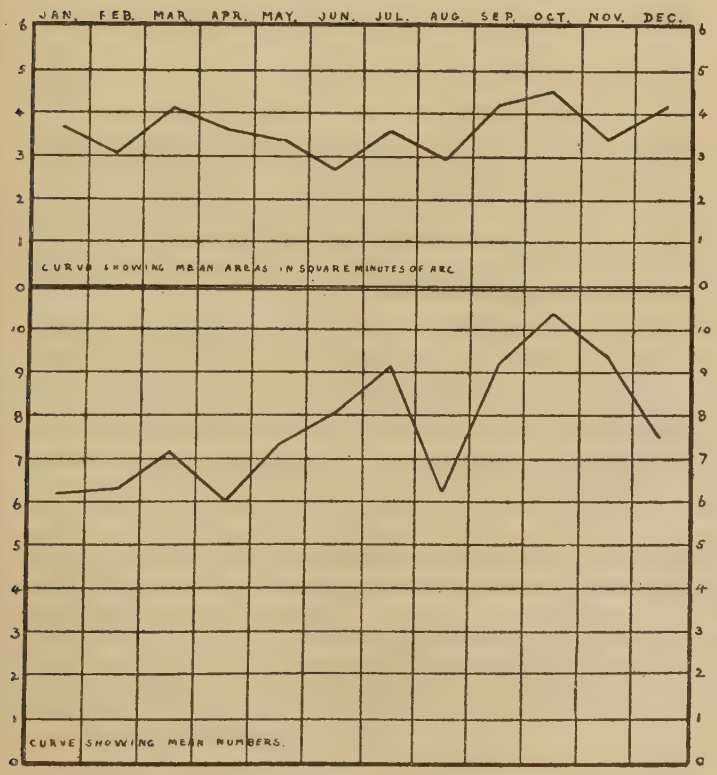

JAN. FEB. MAR. APR. MAY. JUN. JUL. AUG. SEP. OCT. NOV. DEC.
CURVE SHOWING MEAN AREAS IN SQUARE MINUTES OF ARC.
CURVE SHOWING MEAN NUMBERS.

FIG. I.

Table I. gives the means for each month with the number of observations and prominences, which are also shown graphically. The curve showing the mean numbers indicates three periods of increased activity, each of a greater intensity than the preceding one, and each followed by a pronounced decrease. These periods, it will be noticed, follow each other at intervals of from three to four months. Similar periods manifested themselves in previous years, although the intervals are not quite so regular.*

In the case of areas, which are measured in square seconds and converted into square minutes, the first two months show a decline, followed by an increase which coincided with the rise in numbers; then followed a gradual decrease which continued for three months, although numbers were increasing. After June numbers and areas followed each other consistently until November, when their action was again contrary, but in the reverse order, *i.e.* numbers were decreasing and areas increasing. It will be noticed that the greatest activity occurred in October.

TABLE II.

Quadrant.	Prominences.	Per cent.	Mean Density.	Mean Area.	Per cent.
North	420	44·9	3·19	1·64 sq. m.	45·8
South	514	55·0	3·24	1·94 ,, ,,	54·1
East	474	50·7	3·23	1·65 ,, ,,	46·3
West	460	49·2	3·20	1·92 ,, ,,	53·6

This table gives the distribution of prominences, mean densities, and mean areas for each quadrant of the Sun, and shows that activity was predominant in the Southern hemisphere, the percentage for both numbers and areas being very nearly the same. The percentages East show contrary results, in that prominences were a little more numerous on the East limb, but their areas were less. The same feature was observed in the first six months of the year.

As regards density, that was greatest in the South and East. The mean density is arrived at on a scale of 5, where 1 represents a prominence only just visible and 5 one which is very bright, the conditions of seeing being also taken into account.

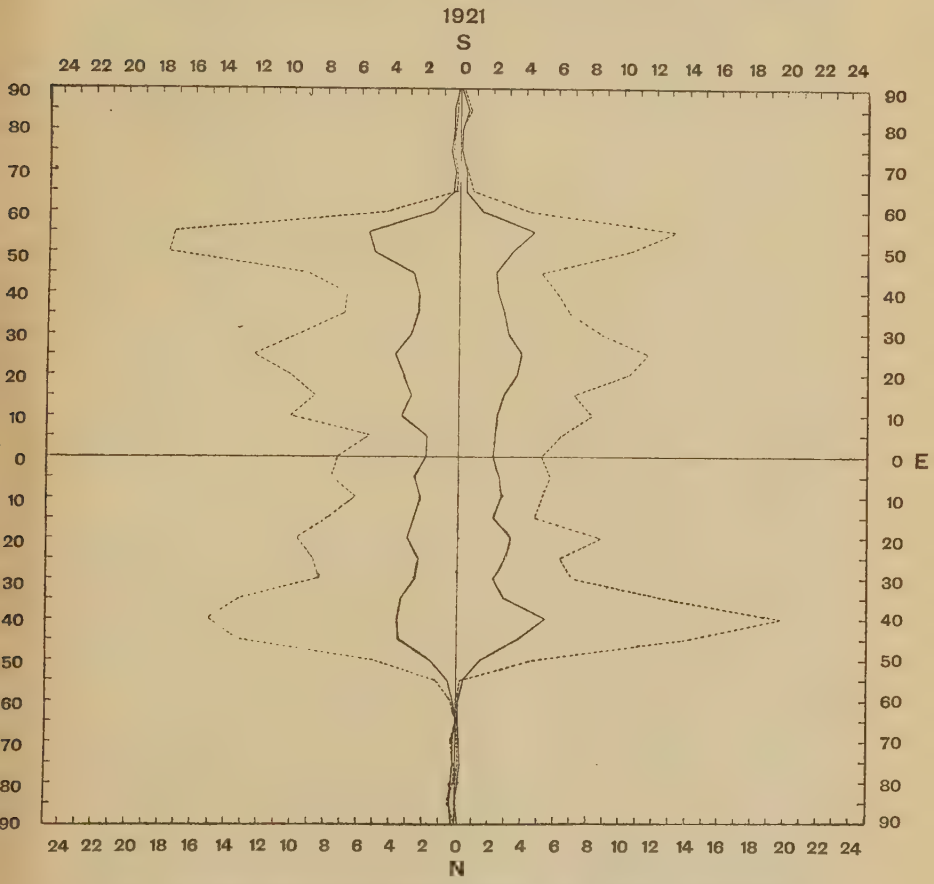
The diagram showing the general distribution of prominences and their areas calls for little comment: the outstanding feature is the pronounced hump at 40° North latitude and 55° South latitude. Considering that we are four years past the maximum, these zones of activity in such high latitudes seem somewhat surprising. Between these two zones activity was more or less uniform, but there are two zones of minor intensity at 20° North latitude and 25° South latitude. There were very few prominences at the poles, and activity may be said to have practically ceased at latitude N. 60° and S. 65°.

The curve for areas follows very closely the general contour of that for numbers. This being the first year in which areas have been measured,

* *B.A.A. Journal*, 29, 192.

no comparison is possible. The curve for areas accentuates the zones of activity previously referred to, and shows very clearly the abrupt cessation of prominences as the poles are approached.

Metallic activity was very much less than in the previous year, only 18 prominences being affected, 13 of which were seen in *b* group and D



Full line = numbers of prominences. Broken line = areas of prominences.

FIG. 2.

lines and 5 in which "1474K" and the red helium lines were seen in addition to *b* and D lines. There was no prominence of great activity; many lines are often seen in these.

Displacements of the H_α line were observed in 13 prominences, 9 of which were to the violet. The greatest displacement was 1·5Å to the violet, which occurred in a small prominence on the West limb on June 11.

Systematic Distribution of Solar Calcium Flocculi. I. *Inclination of Elongated Groups.* By C. P. Butler, A.R.C.Sc. (Plates 8, 9.)

(*Communicated by the Director, Solar Physics Observatory, Cambridge.*)

1. *Introduction.* — Until the application of the spectroheliograph, about 1892, the indicators upon which we were dependent for determinations of solar activity were the spots, faculæ, and prominences; the faculæ being in general observable within a narrow zone near the limb, the prominences at the limb, leaving the dark spots as the only surface phenomena whose changes could be followed from day to day on their passage across the disc.

Of the many features of spot distribution which have been detected from long series of observations, one of the most distinctive is that of the elongated form of the train of small spots which often accompanies moderate or large outbursts.

This is well shown in the early observations by Carrington, and, although it may not be exhibited in every spot outburst, it has been sufficiently frequent to be accepted as a definite characteristic of spot-group formation.

With very few exceptions, the direction of this elongation is equatorward to the west, in either hemisphere, so that the *preceding* extremities of most groups are of *lower* solar latitude than the following portions.

A summary of the inclinations of these spot-group streams has recently been given in a paper by Hale, Ellerman, Nicholson, and Joy (*Ap. Jour.*, 49, 167, 1919). From an investigation of the drawings by Carrington (1856–1861) and Spörer (1861–1893), including 2633 spots extending over $3\frac{1}{2}$ cycles, the mean inclinations of the axes of the spot-groups were found to vary from 0° to 11°, a few individual inclinations up to 14° being recorded. They also found that the amount of the inclination increases with the solar latitude of the group.

With the advent of the spectroheliograph it became possible to examine the relationship between the new features, termed *flocculi*, and the spot-groupings; the general nature of this is that the spots are more or less overlaid by the flocculus matter, and are very often situated at the preceding end of the bright material.

These bright flocculi are frequently clustered in masses of varied shape, and sometimes show evidence of rapid change. An examination of a succession of the records made it apparent that there are a great number of cases where elongation and inclination to the parallels of solar latitude are definitely exhibited by these clustered masses of flocculi.

With very few exceptions, the inclination was found to be in the same sense as that of spot-groups above mentioned—equatorwards to the west in either hemisphere.

When, however, the inclinations were examined in closer detail, the flocculus streams were found to exhibit inclinations from 0° to 40°, with a few exceptional cases showing larger angles. Thus a notable

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difference is found between such masses of flocculi and spot-groups, for the axes of spot-groups show a maximum inclination of only about 14°.

2. *Material Employed for the Investigation.*—The present note summarises some of the results of a preliminary search for system in the distribution of the more marked groups of calcium flocculi. The greater part of the material used for the examination consists of the series of spectroheliograms taken at Kodaikanal in calcium (K_{232}) light, which, by the kindness of the Director of the Kodaikanal Observatory, has been presented to the Solar Physics Observatory since 1906. This consists of an original negative for each available day, with positive copies of plates taken on days when only one negative was obtainable. The scale of these is 60 mm. to the solar diameter. They are received without orientation marks, but with a fiducial line for reference, showing the true horizontal direction.

As the photographs are taken by means of a moving mirror apparatus of the Foucault pattern, the orientation of the field of view varies with the hour-angle of the sun at the time of observation; so that it was necessary to calculate the deviation of the direction of the diurnal motion from a horizontal line on the plate for each record.

For this purpose reduction tables were made by means of Cornu's formula, and the deduced orientation points recorded on the photographic film.

From the negatives, thus oriented, enlarged copies were made on bromide paper, with the mean solar diameter about 193 mm., so as to facilitate comparison with the standard series of photoheliograms used in connection with the spectroheliograms.

3. *Method of Measurement.*—Measurements of the inclinations of the flocculi were first made by taking the positions of the estimated extremities of each flocculus, and plotting these in projection to the equator. This was found unsuitable, and after a few trials the following method proved satisfactory.

It was decided to take the measurements at the time of central meridian passage, when the distortion of the flocculus forms is a minimum. The degree of distortion is not quite constant throughout the year, owing to the inclination of the sun's axis; but, except in a few cases, this introduces no serious difficulty.

A special protractor was obtained from Messrs. Stanley, consisting of two sheets of stout celluloid, pivoted at their common centre. The lower sheet had an engraved position-circle, showing degrees, of diameter to suit the maximum solar diameter on the enlarged paper prints. The upper sheet was provided with a series of parallel lines 1 cm. apart.

It was thus a simple matter to apply this adjustable protractor to the solar photographs, so that the fiducial lines on the prints, indicating the sun's equator and polar axis, coincided with 0° and 90° on the lower position circle. Then the upper plate was turned until the parallel series of lines was estimated to be parallel to the length of the flocculus under examination. The flocculus may be at any distance from the central diameter of the solar disc, and by the above system of parallel

lines the inclination is read off on the position circle by means of the central line.

4. *Selection of Flocculi for Measurement.*—For objects of compact structure the method is quite satisfactory. The chief difficulty encountered has been in the classification of the groups for measurement, as there are many occasions on which the flocculi present are considerably broken up, rendering the process of delimitation of groups a matter requiring much consideration. With all these a critical view has been taken, and if the tendency to elongation was considered as possibly due to a personal selective following of disconnected masses, such groups were rejected for this first examination.

In doubtful cases it was helpful to examine the varying appearance of the groups on successive days, and by this procedure the probability of any unconscious arbitrary selection could be controlled.

5. *Recording the Observations.*—Regarding the actual recording of the observations, it must be noted that the flocculi masses to be dealt with are generally far more irregular in detail than the spots, so that it is more difficult to define what is meant by their position.

As a provisional method, an attempt has been made to estimate the centre of gravity of the area involved in the disturbance, and, in the case of elongations, to take the axis most nearly bisecting the elongated area.

In addition to the measurement of the projected angle of inclination, the position of each flocculus has been recorded in solar latitude and longitude from the central meridian.

For this purpose use has been made of a set of Stonyhurst solar projection discs, copied on photographic film to the standard 193 mm. diameter.

After deciding what is to be measured, the latitude and longitude (from the central meridian) of the preceding end $\left(\frac{\phi w}{\lambda w}\right)$ and the following end $\left(\frac{\phi e}{\lambda e}\right)$ are read off. (w = west = preceding ; e = east = following).

The ratio $\frac{\phi e - \phi w}{\lambda e + \lambda w}$ then gives a measure of the inclination of the stream.

Also, the sign of the difference $(\phi e - \phi w)$ is at once an indication of whether the inclinations are systematic or otherwise. To distinguish readily between inclinations in opposite senses, those trends which slope equatorward to the west have been considered positive, and those in the contrary direction, equatorward to the east, negative inclinations.

6. *Types of Flocculi.*—In order to keep some record of the types of flocculi dealt with, a first rough division into three types has been made, depending on the degree of dispersion of the component parts of the masses. These are designated compact (*c*), dispersed (*d*), and very dispersed (*v.d*).

Also, on account of there being at times flocculi of very great extent in longitude, a rough division has been made into long (L) and short (S); the term short would include all flocculi not longer than about 15°,

corresponding to a little over the travel due to the sun's rotation in one day. This is a useful limitation, as it facilitates the separation of records in cases where several flocculi are present on the central meridian simultaneously.

The appearance as regards elongation and inclination of each flocculus passing central meridian has been thus recorded, and for this purpose it was found convenient to index under six subdivisions, with the accompanying signs for easy reference.

North.	South.	Meaning of Symbol.
\	/	Means *elongation*, and *certain* inclination equatorward to *west*.
—	—	,, ,, ,, almost parallel to equator.
|	|	,, ,, ,, ,, normal ,, ,,
?\	?/	,, ,, ,, *possible* inclination equatorward to *west*.
?/	?\	,, ,, ,, ,, ,, ,, *east*.
*	*	,, *irregular*, of no definite elongation.

7. *Tabulation for Catalogue.*—This scheme of classification proved satisfactory for the years 1915–1920, including the year of maximum activity, 1917.

The work of recording then practically consisted in the making of a catalogue of all flocculi observed on the central meridian, noting the type of each, and the position and inclination of those showing elongation.

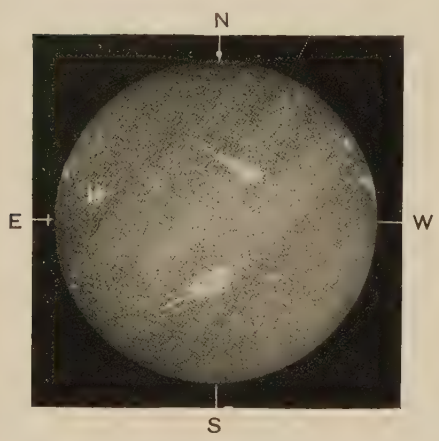
One of the most striking features of this daily log is the evidence furnished concerning the relative variations of activity in the northern and southern hemispheres, which is also a prominent feature of sunspot formation. This is shown in the sample page for May, and is sustained throughout a great part of the cycle.

The following tabulation, for 1917 May, shows the form in which the records were made. At the foot of the table totals are given for the month, showing frequencies of the various types observed. Thus the number under \N, or /S, means that there were 12 and 9 *certain* cases of equatorward-west inclination in the north and south hemisphere respectively. The numbers in the column under ?, giving cases which do not show the general positive trend, are divided into five groups according to the symbols already explained.

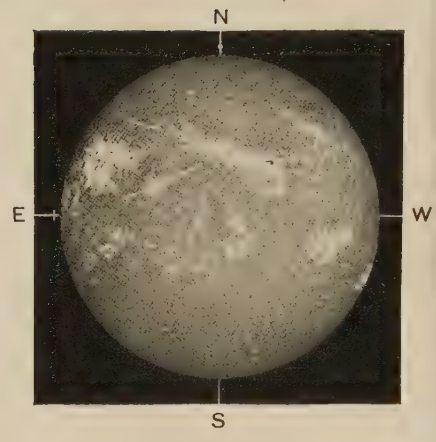
When no *new* feature is present on the central meridian a line is drawn through the space for the day.

I. indicates the inclination measured directly with the protractor.

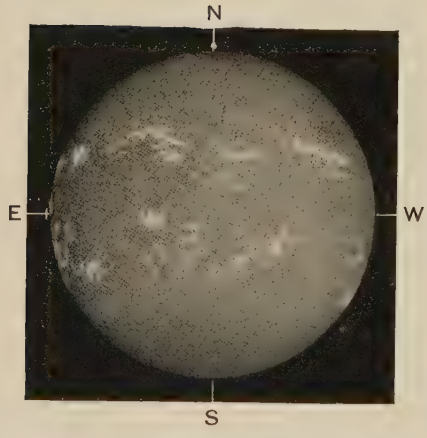
1917.	North. \	?	L.	S.	$\frac{\phi e}{\lambda e}$	$\frac{\phi w}{\lambda w}$	$\frac{\phi e - \phi w}{\lambda e + \lambda w}$	I.	South. /	?	L.	S.	$\frac{\phi e}{\lambda e}$	$\frac{\phi w}{\lambda w}$	$\frac{\phi e - \phi w}{\lambda e + \lambda w}$	I.
May 1		\		Svd					/			Sd	$\frac{-23}{-12\cdot5}$	$\frac{-18\cdot5}{0}$	$\frac{4\cdot5}{12\cdot5}$	22°
2																
3	\			Sc	$\frac{+19\cdot5}{+4\cdot5}$	$\frac{+17\cdot5}{+8\cdot0}$	$\frac{2}{3\cdot5}$	27°		*		Svd				
		*		Sc												
		*		Sc												
4									/			Sc	$\frac{-25\cdot5}{-9}$	$\frac{-21\cdot5}{+1}$	$\frac{4}{10}$	24
5									/		Ld		$\frac{-21}{-18\cdot5}$	$\frac{-11}{+9}$	$\frac{10}{27\cdot5}$	21
6	\			Sd	$\frac{+12}{-3\cdot5}$	$\frac{+11}{+5\cdot5}$	$\frac{1}{9}$	5								
7		*		Svd						*		Svd				
8		*		Sd												
9																
10		—		Sd					/			Sc	$\frac{-14}{-10}$	$\frac{-12}{0}$	$\frac{2}{10}$	13
11	\			Sd	$\frac{+20\cdot5}{-14}$	$\frac{+14\cdot5}{+2\cdot5}$	$\frac{6}{16\cdot5}$	21								
12										/		Svd				
13	\			Sc	$\frac{+25\cdot5}{-4\cdot5}$	$\frac{+23}{+3}$	$\frac{2\cdot5}{7\cdot5}$	17								
14	\			Sc	$\frac{+14}{-1\cdot5}$	$\frac{+6\cdot5}{+11}$	$\frac{7\cdot5}{12\cdot5}$	33	/			Sc	$\frac{-18}{-8}$	$\frac{-16\cdot5}{+3}$	$\frac{1\cdot5}{11}$	9
15	\			Sc	$\frac{+10}{-2}$	$\frac{+8\cdot5}{+8}$	$\frac{1\cdot5}{10}$	10		/		Svd				
16																
17										*		Svd				
18		/	Ld		$\frac{+17\cdot5}{-20\cdot5}$	$\frac{+18\cdot5}{+4\cdot5}$	$\frac{1}{25}$	(-3)	/			Sc	$\frac{-31\cdot5}{-9}$	$\frac{-24\cdot5}{+13}$	$\frac{7}{22}$	19
19	\			Sc	$\frac{+8}{-14\cdot5}$	$\frac{+4}{+1\cdot5}$	$\frac{4}{16}$	13	/			Sd	$\frac{-18\cdot5}{-14\cdot5}$	$\frac{-16\cdot5}{-1\cdot5}$	$\frac{2}{13}$	10
20																
21	\			Sc	$\frac{+11\cdot5}{-10}$	$\frac{+8}{+4}$	$\frac{3\cdot5}{14}$	15								
22																
23																
24																
25	\			Sd	$\frac{+27}{-9\cdot5}$	$\frac{+24\cdot5}{+5}$	$\frac{2\cdot5}{14\cdot5}$	7	/			Sd	$\frac{-21\cdot5}{-1\cdot5}$	$\frac{-20\cdot2}{+6}$	$\frac{1\cdot3}{7\cdot5}$	9
	\			Sd	$\frac{+28}{+5\cdot5}$	$\frac{+19\cdot5}{+12}$	$\frac{8\cdot5}{6\cdot5}$	49								
26																
27										/		Svd				
28		*		Sd						/		Svd				
29		\		Svd												
	\			Svd	$\frac{+6\cdot5}{-3\cdot5}$	$\frac{+5}{+6}$	$\frac{1\cdot5}{9\cdot5}$	6	/			Sd	$\frac{-26\cdot5}{-7\cdot5}$	$\frac{-24}{+10}$	$\frac{2\cdot5}{17\cdot5}$	6
30	\			Svd	$\frac{+17\cdot5}{-2\cdot5}$	$\frac{+15\cdot5}{+13}$	$\frac{2}{15\cdot5}$	7		—		Sd	$\frac{-7}{-7\cdot5}$	$\frac{-6\cdot5}{+3\cdot5}$	$\frac{0\cdot5}{11}$	1
31										*		Svd				
	12 5*								9 4*							
	1—								1—							
	0|								0|						
	2\								4/							
	1/								0\							



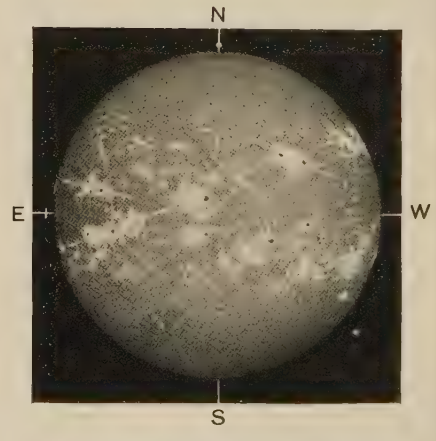
1915 Oct. 20.



1918 April 7.



1918 Feb. 11.



1917 Dec. 27.

Photographs showing General Types of Flocculi.

1915 Oct. 20. N. Typical inclined group.
S. Doubtful group.

1918 April 7. N. Typical long group, separable into parts.
N. Example of negative inclination.
S. Several irregular groups.

1918 Feb. 11. N. } Inclinations shown in long and short groups without spots.
S. }

1917 Dec. 27. N. } Inclinations shown in long and short groups with spots.
S. }

8. *Accuracy of Measurement.*—From the nature of the material it is difficult to see with certainty what accuracy is ultimately attainable, as the chief source of doubt is in the determination of the boundaries of the masses to be measured. They are in general not only extremely irregular and complicated in detail, but at their edges they frequently fade off until lost in the solar réseau.

In working with approximate graphical methods of reduction there is an error of about one degree possible near the days when the Stonyhurst charts are exchanged in steps for variation of the inclination of the sun's axis.

There are also instances when the solar image is not circular; owing to the run of the spectroheliograph being only in a horizontal direction, any alterations in rate may produce ellipticities in various directions when compounded with small errors in the driving gear of the main image-projecting system.

The effect of these combined errors has been partly eliminated by the method of orientation, all comparisons being made on four points, giving a mean position of the centre of the disc for each image.

An effort has been made to make all position readings to half a degree, and from the comparison of consecutive photographs having distinctive small features of apparent stability, it is thought that errors greater than a degree seldom occur in the values of the inclination when the points to be measured have been decided on. A greater range is possible in the process of estimation of the axes of some of the flocculi. This may range from one to two or three degrees, according to the length of the flocculus and its state of aggregation or dispersion.

9. *General Aspect of Appearances on the Disc.*—One of the most prominent features presented by the bright calcium flocculi, next to their main distribution in mid-latitude zones, is the gradually increasing latitude of successive groups appearing over the east limb.

This is particularly noticeable when the disturbances are strong in each hemisphere. In such cases the regular distribution is very marked, and by following up the successive groupings there is evidence of changes of symmetry between north and south. Sometimes the projections of the flocculi trends of each hemisphere meet fairly on the equator. At others the structure is distinctly *en échelon*, the disturbances being alternately stepped behind each other in longitude. The evidence summarised in the present note shows that this pronounced tendency to inclination with respect to the direction of rotation is a sustained feature of the solar calcium flocculi. (See Plate 8.)

10. *Annual Summaries.*—For this preliminary review it will be sufficient to give the results as yearly summaries, made to show the proportions of various types of flocculi exhibited.

In general all the types of flocculi described may be present at all epochs throughout the period under review; that is, we find flocculi which show elongation, some which do not; flocculi which are compact, others which are dispersed to varying degrees.

The chief difference between the yearly summaries appears to be the gradual increase of the proportion of elongated to irregular groups before maximum, and a subsequent preponderance of irregular groups after maximum.

Summary 1915.

	North.						South.					
	Definitely inclined. \	Irregular. *	Parallel. —	Perp. |	? Inclined. ?\	? Negatively inclined. ?/	Definitely inclined. /	Irregular. *	Parallel. —	Perp. |	? Inclined. ?/	? Negatively inclined. ?\
Jan.	8	3	2	0	1	0	0	6	1	0	2	0
Feb.	8	0	2	0	1	0	3	0	1	0	3	0
Mar.	6	3	1	0	1	0	4	3	0	0	3	0
Apr.	4	4	2	0	2	0	3	3	2	0	3	0
May	2	4	5	1	2	1	4	4	1	0	1	0
June	3	2	0	0	4	0	6	1	0	0	0	0
July	4	2	0	0	0	0	11	2	0	0	2	0
Aug.	5	6	1	0	1	0	8	1	1	0	2	0
Sept.	3	1	0	0	5	0	2	0	3	0	4	0
Oct.	11	2	0	0	2	0	3	7	1	0	2	0
Nov.	8	4	1	0	6	0	5	2	1	0	0	0
Dec.	4	4	1	0	2	0	3	2	2	0	0	0
Totals	66	35	15	1	27	1	52	31	13	0	22	0
			145						118			

263 for 1915

Inclined to equator W. / Total examined $\frac{\backslash+?\backslash}{145}=\frac{66+27}{145}=\frac{93}{145}=0{\cdot}64$

Elongated / Total examined $\frac{\backslash+?\backslash+-+|+?/}{145}=\frac{110}{145}=0{\cdot}76$

Probably inclined \ / Total elongated $\frac{93}{110}=0{\cdot}85$

Certainly inclined \ / Total elongated $\frac{66}{110}=0{\cdot}60$

Inclined to equator W. / Total examined $\frac{/+?/}{118}=\frac{52+22}{118}=\frac{74}{118}=0{\cdot}63$

Elongated / Total examined $\frac{/+?/+-+|+?\backslash}{118}=\frac{87}{118}=0{\cdot}74$

Probably inclined / / Total elongated $\frac{74}{87}=0{\cdot}85$

Certainly inclined / / Total elongated $\frac{52}{87}=0{\cdot}60$

Summary 1916.

	North.						South.					
	\	*	—	|	?\	?/	/	*	—	|	?/	?\
Jan.	9	6	2	0	3	0	2	4	1	0	3	0
Feb.	3	7	4	0	3	0	3	5	0	0	4	0
Mar.	4	5	6	0	2	0	5	5	1	0	4	0
Apr.	7	5	1	0	1	0	5	5	3	0	2	0
May	7	3	2	1	1	1	8	1	2	0	1	1
June	7	1	3	0	1	1	7	3	0	0	0	0
July	12	2	0	0	0	0	5	5	0	0	6	0
Aug.	4	2	1	1	3	0	1	4	1	1	1	1
Sept.	5	3	1	2	3	0	6	4	0	0	3	0
Oct.	8	1	2	0	0	0	4	6	3	0	1	0
Nov.	9	6	3	0	0	0	6	2	1	0	1	0
Dec.	6	6	1	0	4	1	9	4	1	0	1	0
Totals	81	48	26	4	21	3	61	48	13	1	27	2
	183						152					

335 for 1916

Inclined to equator W. / Total examined	$\frac{\backslash+?\backslash}{183}$	$\frac{81+21}{183}=\frac{102}{183}$	$=0{\cdot}56$
Elongated / Total examined	$\frac{\backslash+?\backslash+-+\mid+?/}{183}$	$\frac{135}{183}$	$=0{\cdot}74$
Probably inclined \+?\ / Total elongated		$\frac{102}{135}$	$=0{\cdot}76$
Certainly inclined \ / Total elongated		$\frac{81}{135}$	$=0{\cdot}60$

Inclined to equator W. / Total examined	$\frac{/+?/}{152}$	$\frac{61+27}{152}=\frac{88}{152}$	$=0{\cdot}58$
Elongated / Total examined	$\frac{/+?/+-+\mid+?\backslash}{152}$	$\frac{104}{152}$	$=0{\cdot}68$
Probably inclined /+?/ / Total elongated		$\frac{88}{104}$	$=0{\cdot}85$
Certainly inclined / / Total elongated		$\frac{61}{104}$	$=0{\cdot}59$

Summary 1917.

	North.						South.					
	\	*	—	|	?\	?/	/	*	—	|	?/	?\
Jan.	5	6	6	0	0	0	4	10	1	0	1	0
Feb.	9	3	1	0	1	1	7	3	2	0	2	0
Mar.	8	10	6	0	2	0	5	2	3	0	1	0
Apr.	7	7	2	0	4	0	5	6	1	0	3	0
May	12	5	1	0	2	1	9	4	1	0	4	0
June	9	4	5	0	7	0	10	5	2	0	4	0
July	10	6	0	0	4	0	6	6	1	0	4	0
Aug.	11	3	1	1	3	0	10	10	2	0	3	0
Sept.	6	5	4	0	1	1	4	9	3	0	1	0
Oct.	11	9	2	0	1	0	8	8	3	0	0	0
Nov.	9	4	3	0	2	1	8	6	0	0	1	0
Dec.	9	7	2	0	6	1	7	9	0	0	3	3
Totals	106	69	33	1	33	5	83	78	19	0	27	3
	247						210					

457 for 1917

Inclined to equator W. / Total examined $\frac{\backslash+?\backslash}{247}=\frac{106+33}{247}=\frac{139}{247}=0{\cdot}56$

Elongated / Total examined $\frac{\backslash+?\backslash+-+|+?/}{247}=\frac{178}{247}=0{\cdot}72$

Probably inclined \+?\ / Total elongated $\frac{139}{178}=0{\cdot}78$

Certainly inclined \ / Total elongated $\frac{106}{178}=0{\cdot}60$

Inclined to equator W. / Total examined $\frac{/+?/}{210}=\frac{83+27}{210}=\frac{110}{210}=0{\cdot}52$

Elongated / Total examined $\frac{/+?/+-+|+?\backslash}{210}=\frac{132}{210}=0{\cdot}63$

Probably inclined /+?/ / Total elongated $\frac{110}{132}=0{\cdot}83$

Certainly inclined / / Total elongated $\frac{83}{132}=0{\cdot}63$

Summary 1918.

	North.						South.					
	\	*	—	|	?\	?/	/	*	—	|	?/	?\
Jan.	6	6	1	0	4	0	9	4	1	0	2	0
Feb.	6	5	3	0	4	0	9	5	0	0	2	1
Mar.	0	16	4	0	1	1	3	11	3	0	3	1
Apr.	5	6	2	0	5	1	5	5	2	1	2	0
May	4	7	2	0	4	0	3	5	0	0	4	0
June	5	12	1	0	5	0	3	12	0	0	2	1
July	5	7	4	1	1	0	6	5	3	0	3	0
Aug.	3	19	1	1	2	0	5	10	2	0	0	0
Sept.	4	9	1	0	2	0	4	9	4	0	2	1
Oct.	0	16	5	0	2	0	0	23	4	0	1	1
Nov.	4	20	0	0	1	0	1	16	1	0	3	0
Dec.	2	15	0	0	0	0	4	13	2	0	0	0
Totals	44	138	24	2	31	2	52	118	22	1	24	5

241 222

463 for 1918

Inclined to equator W. / Total examined	$\frac{\backslash+?\backslash}{241}$	$=\frac{44+31}{241}=\frac{75}{241}=0{\cdot}31$	Inclined to equator W. / Total examined	$\frac{/+?/}{222}$	$=\frac{52+24}{222}=\frac{76}{222}=0{\cdot}34$
Elongated / Total examined	$\frac{\backslash+?\backslash+-+\|+?/}{241}$	$=\frac{103}{241}=0{\cdot}43$	Elongated / Total examined	$\frac{/+?/+-+\|+?\backslash}{222}$	$=\frac{104}{222}=0{\cdot}47$
Probably inclined \+?\ / Total elongated		$\frac{75}{103}=0{\cdot}73$	Probably inclined /+?/ / Total elongated		$\frac{76}{104}=0{\cdot}73$
Certainly inclined \ / Total elongated		$\frac{44}{103}=0{\cdot}43$	Certainly inclined / / Total elongated		$\frac{52}{104}=0{\cdot}50$

Summary 1919.

	North.						South.					
	\	*	—	|	?\	?/	/	*	—	|	?/	?\
Jan.	4	15	1	0	2	0	2	20	1	0	0	1
Feb.	4	12	2	0	1	0	1	11	2	0	3	0
Mar.	7	13	1	0	2	0	3	14	1	0	2	1
Apr.	1	13	1	0	3	0	2	8	3	0	2	0
May	1	14	0	0	7	0	4	9	2	0	3	0
June	7	9	1	2	1	0	7	9	0	0	4	2
July	4	13	2	0	3	0	3	9	1	0	3	0
Aug.	3	16	0	0	2	0	2	15	3	0	4	0
Sept.	2	9	0	0	3	0	3	13	3	0	2	0
Oct.	2	6	2	0	1	1	6	13	1	0	2	0
Nov.	3	6	3	0	2	0	3	10	1	0	1	0
Dec.	5	9	3	0	0	1	3	12	0	0	0	0
Totals	43	135	16	2	27	2	39	143	18	0	26	4
				225						230		

455 for 1919

Inclined to equator W. / Total examined $\frac{\backslash+?\backslash}{225}=\frac{43+27}{225}=\frac{70}{225}=0{\cdot}31$

Elongated / Total examined $\frac{\backslash+?\backslash+-+|+?/}{225}=\frac{90}{225}=0{\cdot}40$

Probably inclined \ / Total elongated $\frac{70}{90}=0{\cdot}78$

Certainly inclined \ / Total elongated $\frac{43}{90}=0{\cdot}48$

Inclined to equator W. / Total examined $\frac{/+?/}{230}=\frac{39+26}{230}=\frac{65}{230}=0{\cdot}28$

Elongated / Total examined $\frac{/+?/+-+|+?\backslash}{230}=\frac{87}{230}=0{\cdot}38$

Probably inclined / / Total elongated $\frac{65}{87}=0{\cdot}75$

Certainly inclined / / Total elongated $\frac{39}{87}=0{\cdot}45$

Summary 1920.

	North.						South.					
	\	*	—	|	?\	?/	/	*	—	|	?/	?\
Jan.	3	10	1	0	0	0	4	2	1	1	0	0
Feb.	1	9	2	0	0	1		6	1	0	1	1
Mar.	4	15	0	0	0	0	3	14	2	0	4	0
Apr.	1	12	2	0	1	0	0	11	1	0	1	0
May	4	13	2	0	2	1	4	10	0	0	2	0
June	4	12	1	0	2	1	1	6	1	0	2	0
July	0	17	1	0	2	0	1	12	4	0	0	0
Aug.	0	13	0	0	3	0	0	11	1	0	0	0
Sept.	3	10	1	0	1	0	2	5	1	0	2	0
Oct.	5	7	0	0	0	0	4	3	0	0	1	1
Nov.	0	8	0	0	4	0	2	1	0	0	5	0
Dec.	1	12	1	0	2	1	2	11	0	0	4	1
Totals	26	138	11	0	17	4	26	92	12	1	22	3
			196						156			

352 for 1920

Inclined to equator W. / Total examined $= \frac{\backslash + ? \backslash}{196} = \frac{26+17}{196} = \frac{43}{196} = 0{\cdot}22$

Elongated / Total examined $= \frac{\backslash + ? \backslash + - + | + ? /}{196} = \frac{58}{196} = 0{\cdot}30$

Probably inclined $\backslash + ? \backslash$ / Total elongated $= \frac{43}{58} = 0{\cdot}74$

Certainly inclined $\backslash$ / Total elongated $= \frac{26}{58} = 0{\cdot}45$

Inclined to equator W. / Total examined $= \frac{/ + ? /}{156} = \frac{26+22}{156} = \frac{48}{156} = 0{\cdot}31$

Elongated / Total examined $= \frac{/ + ? / + - + | + ? \backslash}{156} = \frac{64}{156} = 0{\cdot}41$

Probably inclined $/ + ? /$ / Total elongated $= \frac{48}{64} = 0{\cdot}75$

Certainly inclined / / Total elongated $= \frac{26}{64} = 0{\cdot}41$

Summary for Six Years (1915–1920).

Year.	North. Total North.	\	*	—	|	?\	?/	South. Total South.	/	*	—	|	?/	?\	Total for Year.
1915	145	66	35	15	1	27	1	118	52	31	13	0	22	0	263
1916	183	81	48	26	4	21	3	152	61	48	13	1	27	2	335
1917	247	106	69	33	1	33	5	210	83	78	19	0	27	3	457
1918	241	44	138	24	2	31	2	222	52	118	22	1	24	5	463
1919	225	43	135	16	2	27	2	230	39	143	18	0	26	4	455
1920	196	26	138	11	0	17	4	156	26	92	12	1	22	3	352
Totals for six years	1237	366	563	125	10	156	17	1088	313	510	97	3	148	17	2325

Inclined to equator W. / Total examined $\frac{\backslash+?\backslash}{1237}=\frac{366+156}{1237}=\frac{522}{1237}=0\cdot42$

Elongated / Total examined $\frac{\backslash+?\backslash+-+|+?/}{1237}=\frac{674}{1237}=0\cdot55$

Probably inclined \ / Total elongated $\frac{522}{674}=0\cdot78$

Certainly inclined \ / Total elongated $\frac{366}{674}=0\cdot54$

Inclined to equator W. / Total examined $\frac{/+?/}{1088}=\frac{313+148}{1088}=\frac{461}{1088}=0\cdot42$

Elongated / Total examined $\frac{/+?/+-+|+?\backslash}{1088}=\frac{578}{1088}=0\cdot53$

Probably inclined / / Total elongated $=\frac{461}{578}=0\cdot80$

Certainly inclined / / Total elongated $=\frac{313}{578}=0\cdot54$

Comparing the seven tables, we notice

(1) The close relationship of the variation in type of the flocculi for the two hemispheres, as shown by the general similarity in the numerals under the several headings.

(2) The extreme smallness of the numbers of flocculi showing inclinations of 90° or negative inclinations. This feature is sustained throughout the six years.

(3) The marked variations of the relative proportions of different types of flocculi at different epochs.

(4) The remarkable regularity exhibited by the relative percentages of the different groups in the two hemispheres. This is indicated in the following table showing percentages for the six years.

Percentages of Occurrence of Four Classes in Yearly Tables.

	North.							South.						
	1915.	1916.	1917.	1918.	1919.	1920.	1915 to 1920.	1915.	1916.	1917.	1918.	1919.	1920.	1915 to 1920.
Inclined to equator W. / Total examined	64	56	56	31	31	22	42	63	58	52	34	28	31	42
Elongated / Total examined	76	74	72	43	40	30	55	74	68	63	47	38	41	53
Probably inclined equator W. / Total elongated	85	76	78	73	78	74	78	85	85	83	73	75	75	80
Certainly inclined equator W. / Total elongated	60	60	60	43	48	45	54	60	59	63	50	45	41	54

11. *Range of Inclinations and Distribution of Frequency of Occurrence within the Range.*—In the endeavour to interpret these observations one is led to inquire into the distribution on the solar surface with respect to

(1) The direction of the sun's rotation.
(2) Latitude.
(3) Variations throughout the cycle of solar activity.

In order to determine whether there may be any specially favoured inclinations the individual angles were plotted in the form of fans radiating from a single centre, frequency being shown by the length of the radius. The range of angles is somewhat wide, but there are clear signs of concentration into groups about certain directions. The accompanying diagram shows the angles plotted in zones of 5° for the six years, and the selective grouping is less marked (see fig. 1).

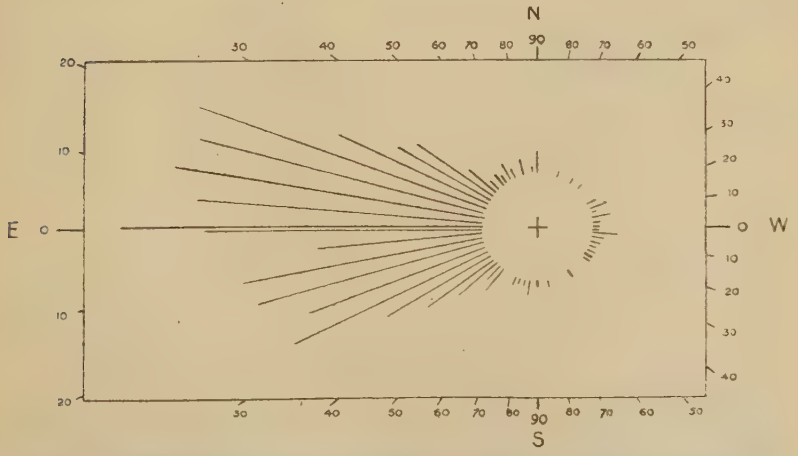


FIG. 1.—Inclinations of Flocculi Axes for six years, 1915–1920, grouped in zones of 5°. (This grouping somewhat masks the indication of favoured angles which is shown on the plottings of individual angles.)

Inclinations similar in amount to those found in spot-groups (0° to 11°) are clearly shown, but there are also obvious signs of great frequency of inclinations of higher value, and some slight indication

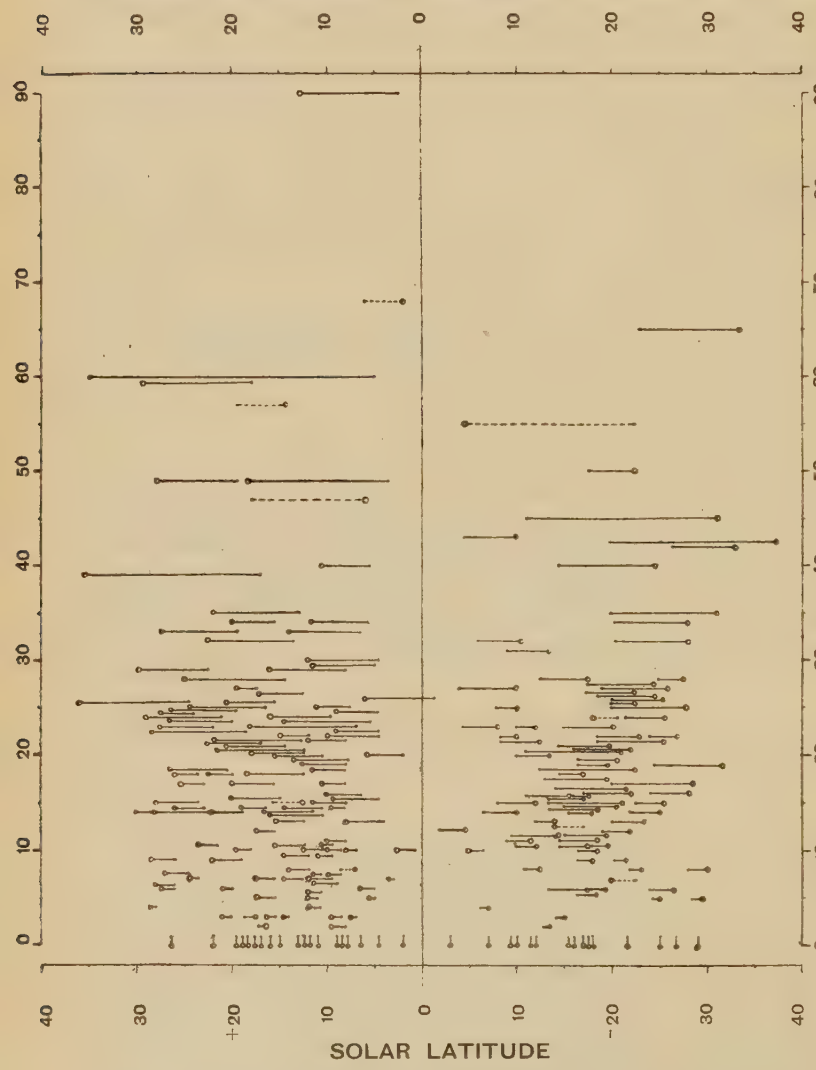

40 30 20 10 0 10 20 30 40
0 10 20 30 40 50 60 70 80 90
40 30 +20 10 0 10 -20 30 40
SOLAR LATITUDE
→ Inclination of flocculi axes.
• = preceding end of flocculus.
o = following ,,
o—• = flocculus with *positive* inclination, equatorward to *west*.
o·····• = ,, ,, *negative* ,, ,, *east*.

FIG. 2.—Variation of Inclination with Solar Latitude (1917).

of clustering about certain favoured values, about 15°, 21°, 28°–32°, shown almost equally well in northern and southern latitudes. The observational data available are not yet sufficient to disentangle the systematic tendency which the clustering about these favoured directions indicates.

Negative Inclinations.—The number of negative inclinations showing trends equatorward to the *east* is remarkably small, amounting to only 34 cases out of 2325 flocculi examined, of which 1252 showed elongation.

In some cases these may be due to changes in form of a complicated flocculus of a larger scale, which has been divided into smaller portions for record.

There are, however, a small number of isolated cases, free from such interpretation, in which we find a compact flocculus showing inclination in the opposite way to that found for the general masses of flocculi.

12. *Inclination and Latitude.*—The accompanying diagram (see fig. 2) shows, for the year 1917, the variation of inclination with the positions of the flocculi in latitude, the preceding end of each flocculus being indicated by a dot, ●, and the following end by a small circle, ○.

From the occurrence of similar inclinations over wide ranges of latitude, it appears that there is not the same variation with latitude which is shown in the axes of spot-groups. Tabulations have been made showing the inclinations found in zones of 5° in latitude, from which the mean inclination in each zone has been extracted. These values for the six years are shown in the accompanying table. The small numbers in parentheses indicate the number of flocculi in each case.

Mean Inclinations for 5° Zones of Latitude.

φ	Inclination. 1915. N.	1915. S.	1916. N.	1916. S.	1917. N.	1917. S.	1918. N.	1918. S.	1919. N.	1919. S.	1920. N.	1920. S.	Mean Inclination for Six Years.
°	°	°	°	°	°	°	°	°	°	°	°	°	°
40	20(2)	10(1)	7·5(2)	11(1)									12·7(6)
35	21(4)	18·5(2)	24·5(2)	10(1)	25(1)	42(1)		10(1)	18(1)			37(1)	22·3(14)
30	18·6(14)	23·3(6)	23·9(12)	16·1(7)	15·5(16)	19(14)	13·3(3)	16·6(5)	15·5(2)				18·6(79)
25	16·9(20)	17(29)	21·1(21)	23(26)	23·6(19)	22·7(22)	18·5(6)	22(16)	13·6(3)	21(7)	3·5(2)	15·7(4)	20·2(175)
20	15·5(24)	14·3(15)	20·2(22)	18·7(35)	10·3(36)	11·5(36)	16(19)	13·8(16)	21·3(9)	13·5(16)	6·5(11)	26·2(10)	15·1(243)
15	27·5(4)	0(1)	17·1(34)	17·9(9)	11·1(37)	7·0(16)	12·2(15)	12·7(21)	9·8(23)	7·7(12)	11·7(10)	15·3(7)	12·6(189)
10	12·3(7)		20·6(11)	2(1)	15·1(29)	19·1(10)	4·6(14)	5·3(9)	7·3(15)	9·1(21)	10·6(13)	19·2(14)	12·2(144)
5	20(1)		6·6(3)		−1·3(6)	6(2)	15·1(8)	4(1)	9(7)	8(1)	6·5(2)	−9·6(3)	6·6(34)
0													

Thus for a certain range in latitude 0°–25°, the variation shows some slight indication of increase of mean inclination with latitude, similar to the spot-group axes. Beyond this there is irregularity.

The variations are also clearly shown in the accompanying chart (see fig. 3), giving the actual projected positions of tracks for 1917 on the sun's surface, between the longitudes 100° and 300°. It serves to show a very disturbed region between longitudes 110° and 190°, whilst the rest of the region gives a good idea of the more typical distribution of inclinations which has generally been observed during the six years of observation.

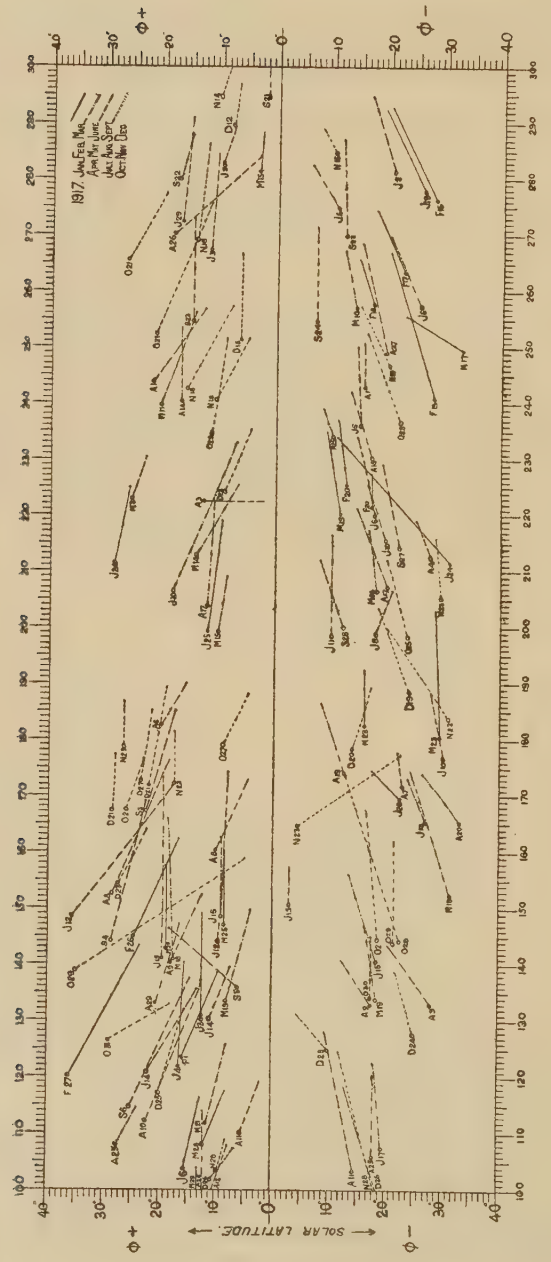


FIG. 3.—Diagram showing projected tracks in actual positions on solar surface, 1917. Selected region from solar longitude 100°–300°, including the disturbed region from 110° to 190°, persisting throughout the year.

13. *Variation of Inclination during Cycle of Solar Activity.*—The individual inclinations for the six years 1915–1920 have been plotted on a time scale (see fig. 3), and certain features appear to be presented throughout the period examined.

(1) The occurrence of inclinations much greater than the limits found for spot-groups (0°–14°).
(2) Indications of periodicity in the occurrence of the higher inclinations.
(3) Some slight indication that these variations of inclination alternate in the two hemispheres.

14. *Relationship between Flocculi of Different Types.*—Evidence having been found of a tendency to systematic inclination in elongated masses of flocculi, the relationship of the smaller masses to the larger aggregations, which are frequently met with at different epochs in the solar-spot cycle, has been examined.

One of the difficulties lies in the not infrequent dissolving of obvious masses of flocculi into such sparse groups that they can only just be distinguished from the details of the general solar réseau.

Taking all the flocculi observed, it is found that there is a considerable number so scattered that when attempt is made to divide them into parts of smaller extent than about 15°, it is difficult to find, from a single appearance on the central meridian, any definite sign of elongation.

Since, however, as mentioned in § 15, the life of flocculi in general seems to be long compared with that of spots, it may be that many of these very scattered groups are the survivors of earlier more compact groups, or the forerunners of later ones. Examples of formations where the life-history is certain have been noted, and in some cases this transition from compact to scattered masses is quite definite. This affords evidence that a mass of sparse flocculi may often be regarded as a definite group.

If this be taken as granted, a considerable number of these dispersed aggregations show the same tendency to elongation as is exhibited by the smaller masses followed from birth.

To give an idea of the frequency of occurrence of these scattered types of flocculi, those observed during the six years under discussion have been divided into two classes, according as they appeared (*a*) definitely separable into smaller groups; (*b*) non-separable. These are shown in the following table:—

Divisibility of Long Streams.

	1915.		1916.		1917.		1918.		1919.		1920.	
	N.	S.	N.	S.	N.	S.	N.	S.	N.	S.	N.	S.
(*a*) Separable . .	20	13	15	12	22	21	24	16	12	17	11	20
(*b*) Non-separable . .	4	2	3	6	17	6	4	1	9	4	8	3

Discrimination between such cases is a matter of varying degrees of difficulty. In many cases it is of great assistance to note the latitudes of each of the suspected component parts. If these latitudes remain

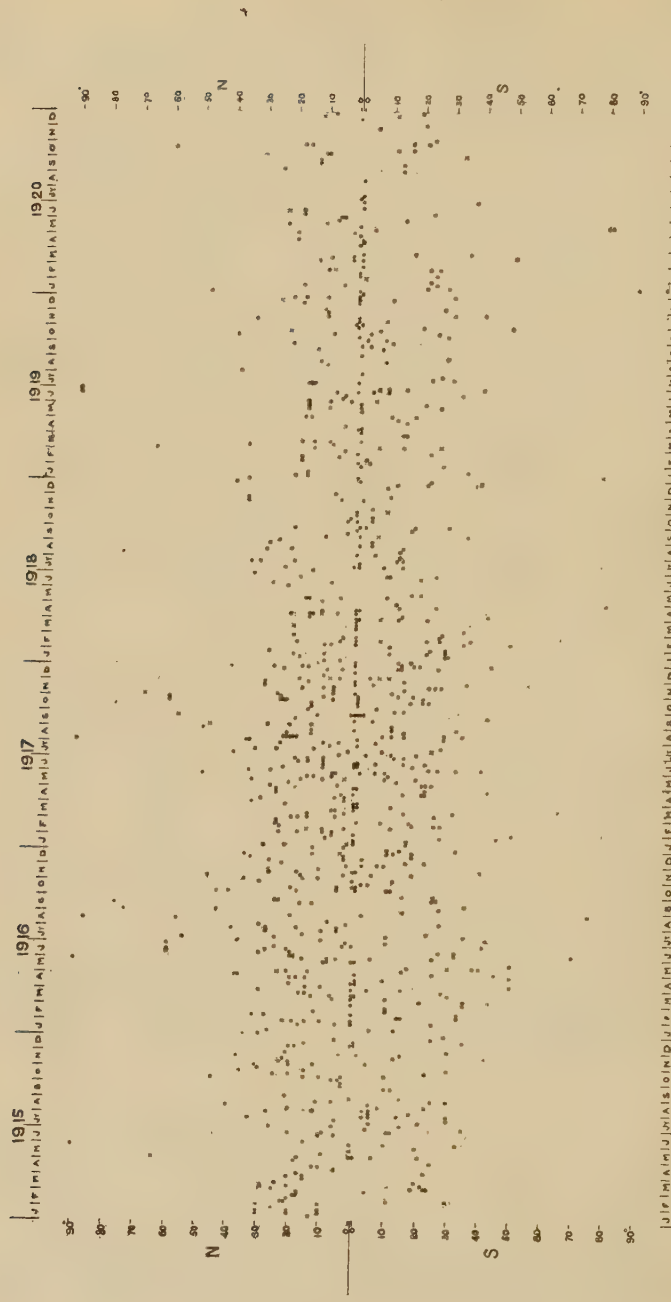
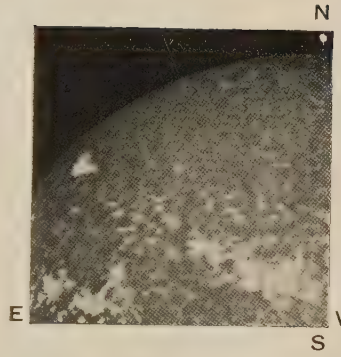
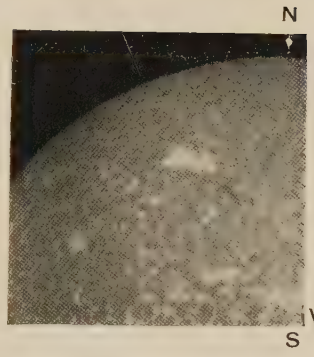


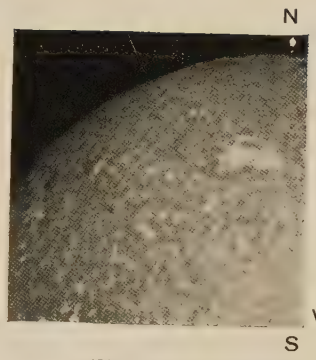
FIG. 4.—Diagram showing Inclination observed during the period 1915–1920.



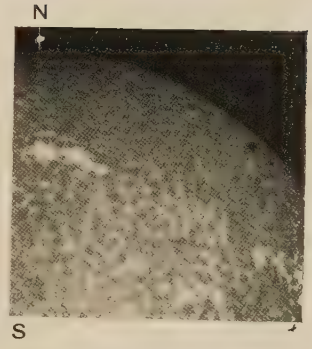
1916 March 10.



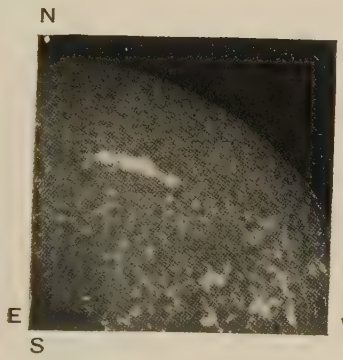
1916 March 13.



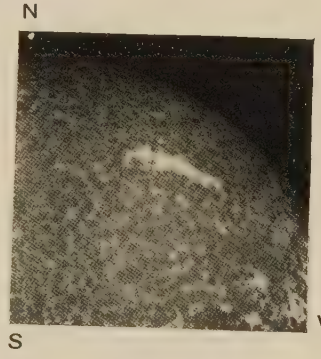
1916 March 14.



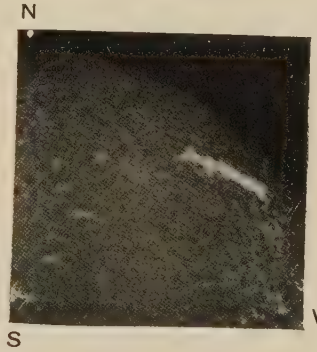
1916 March 15.



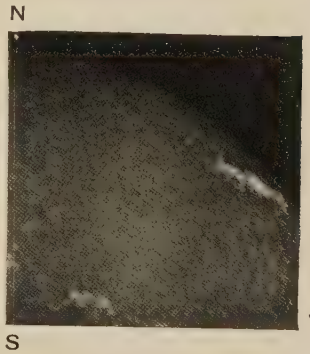
1916 March 16.



1916 March 17.



1916 March 18.



1916 March 19.

Photographs showing Development of Small Flocculus, 1916 March 10–19.

N = Sun's North Pole.

constant on successive days they have been treated as separate masses, and not as details of a long stream of flocculi. When, however, the latitudes vary continuously along the stream they have been ascribed to a single long stream.

15. *Life-History of Flocculi.*—A notable feature of the life-history of flocculi, as contrasted with that of spots, is their greater longevity. This question will be dealt with more fully later, but it may be mentioned here that only few flocculi are found with lives less than the full thirteen days or more of transit across the disc; in other words, of the flocculi which appear at the eastern limb, by far the greater number last until they pass over the western limb a fortnight later.

In most of the cases of observed births on the disc, the resulting flocculi survive until they pass round the western limb.

These flocculi observed soon after birth, especially the smaller ones, furnish important data respecting the inclination phenomenon, as we can then trace the development with some certainty. All cases so far recorded are being investigated, and a notable case of rapid growth of elongation is given in the accompanying illustration (Plate 9), showing the growth in a selective direction of a small flocculus from 1916 March 10–19. Details of the changes observed are given in the following table:—

Development of a Small Calcium Flocculus: 1916 *March* 10–19.

ϕe and ϕw are latitudes of *following* and *preceding* extremities respectively.

λe and λw are longitudes from meridian of *following* and *preceding* extremities respectively.

	$\frac{\phi e}{\lambda e}$	$\frac{\phi w}{\lambda w}$	$\frac{\phi e - \phi w}{\lambda e + \lambda w}$	
1916 Mar. 9	$+40°$			Just on limb, longitude doubtful.
10	$\frac{+40}{-70}$	$\frac{+38}{-57}$	$\frac{2}{13}$	
11				Poor plate.
12	$\frac{+40}{-40}$	$\frac{+37}{-27}$	$\frac{3}{13}$	
13	$\frac{+40}{-27}$	$\frac{+37}{-15}$	$\frac{3}{12}$	
14	$\frac{+39\cdot5}{-13\cdot5}$	$\frac{+36\cdot5}{-2}$	$\frac{3}{11\cdot5}$	
15	$\frac{+40}{-1\cdot5}$	$\frac{+35}{+13\cdot5}$	$\frac{5}{15}$	
16	$\frac{+40}{+10}$	$\frac{+34\cdot5}{+28}$	$\frac{5\cdot5}{18}$	
17	$\frac{+40}{+21\cdot5}$	$\frac{+34}{+42\cdot5}$	$\frac{6}{21}$	
18	$\frac{+40\cdot5}{+38}$	$\frac{+34}{+60}$	$\frac{6\cdot5}{22}$	
19	$\frac{+40}{+50}$	$\frac{+33?}{+80}$	$\frac{7}{30}$	Western end on limb.

Omitting the days when measurements are doubtful owing to proximity to the limb, we then have :—

From March 10–18.	Following End.	Preceding End.	Differential Motion between Preceding and Following Ends.
Change in latitude	0°	4°	4°
Change in longitude from meridian	108°	117°	9°

There are very few cases of flocculi observable at both birth and death, and these only by including flocculi so sparse as to be barely distinguishable from the general solar réseau.

Recurrence.—In judging of recurrences, we are met with two difficulties additional to those encountered in the recognition of recurrences of sunspots. *First*, the areas involved are so much larger that the recognition of any fiducial point becomes more uncertain than in the case of smaller spot areas. *Second*, the changes in the detailed forms of the flocculi are so great that it is not easy to be certain of true recurrence after the fourteen days of invisibility.

Nevertheless there are many cases of flocculi with distinctive features rendering it possible to recognise their return with confidence. Many of these are found to retain an inclination similar to that exhibited during their first appearance.

16. *Connection with Determinations of Solar Rotation.*—It is easy to see that the effect of latitude variation of the period of rotation is far too small to be a direct influence in the production of the large inclinations observed. Sometimes masses of flocculi are found showing large inclinations a few days after birth, whereas it would take more than a whole rotation-period to develop such angles by differential drift due to rotation. This has also been noted by Royds (see § 17) with regard to H_α filaments.

Measurements of many flocculi show that the inclination of a group, once formed, appears to be remarkably constant throughout its passage across the disc, and instances have also been observed of almost constant angle in cases of undoubted recurrents.

From the general systematic character of these flocculi formations it would thus appear that there is some movement which is superimposed on the general rotation.

It has been for many years accepted that the faculæ rotate faster than either spots or the absorbing vapours giving rise to the Fraunhofer spectrum. Although, in dealing with spectroheliograms, we believe we are investigating specially localised layers, depending on the radial extension of the vapours giving the monochromatic light utilised, the forms of flocculi are so similar to those of faculæ observed simultaneously that there can be little doubt of their close association.

If a movement of gaseous matter is involved, changeable in direction and amount at different times, it might affect the movement of such lower layers as are utilised for the spectroscopic observations of solar rotation.

17. *Relation between Bright and Dark Calcium Flocculi.*—Soon after the commencement of the present investigation in 1918 it was

recognised that the dark calcium flocculi (filaments) had some systematic relationship to the bright flocculi. The dark filaments appear to be remarkably consistent in their ramification *between* the main bright masses, thus exhibiting, in cases where they are sufficiently regular, similar inclinations from 0° to 90°, and in the same sense, equatorward to the west in either hemisphere. While, however, the bright flocculi are, in general, large clustered masses, although often elongated, the dark filaments are thin and sinuous.

In August 1920, a paper was published by T. Royds on "Some features of dark H_α markings on the Sun" (*Kodaikanal Observatory Bulletin*, 63), in which he summarises his measurements of the inclinations of such markings. Although he deals with hydrogen in place of calcium, it seems evident that the appearances in the dark markings in each case represent a similar distribution of the mixed materials in the sun's upper atmosphere.

On the calcium photographs the dark feature often first appears as a slightly curved filament extending across the solar equator; then it gradually lengthens out, leaving the outstanding wings at the following ends in northern or southern latitudes, or possibly in both, inclined at moderate angles to the equator. It is not always possible to follow all the transition stages, but there are many records of this appearance spread over wide intervals of time.

It is only the most prominent cases which are well shown on calcium spectroheliograms taken with a combination of K_{232} light. The dark filaments are best shown on plates taken with the centre K_3 component alone, and this necessitates an instrument with large dispersion.

The present note attempts to summarise certain features in the forms of flocculi observed, for specific reasons, chiefly on the central meridian.

But in the examination of the available data the observer has had to examine the sequence of forms in other positions, and has gained the strong impression that the observed elongation is a result of directional streaming in the strata where the flocculi float.

The investigation is now to be turned to an examination of the changes in position of the ends of the elongated masses. This is the obvious method to be employed for elucidating the question whether the phenomenon are to be explained as due primarily to eruptive action, or as the much more probable indication of a life-history in which the flocculus is the primary phenomenon which may in its downward motion result in a sunspot.

Summary.

1. From an examination of spectroheliograms taken with monochromatic calcium light (K_{232}), for the period 1915–1920, it has been found that a characteristic feature of a considerable proportion of the clustered masses of solar flocculi is their tendency to become elongated in a specific direction; that is, inclined equatorward to the west in either hemisphere.

2. The chief difference shown by the yearly summaries appears to be the gradual increase of the proportion of elongated to irregular

groups before maximum, and a subsequent predominance of irregular groups after maximum.

3. The inclinations measured from the parallels of latitude range in general from 0° to 40°, with a few cases of specially high inclination up to even 90°. The range is thus considerably greater than that previously recognised in the axes of spot-groups.

4. The high-angle trends appear to occur over a wide range of latitude, thus differing from the regular increase with latitude shown by axes of spot-groups.

5. The inclinations observed are not uniformly distributed from 0° to 40°, but show indications of maximum frequencies about certain values, in the neighbourhood of 15°, 21°, 28°–32°.

6. There appears to be a periodicity in time in the alternation of high and medium inclinations of the trends, with some evidence of disagreement of phase in the two hemispheres.

7. Apart from the variation in total numbers connected with changes in activity, the above features 1, 3, 4, 5, 6 are sustained throughout the six years under observation, which include 1917, the year of the last maximum.

I am indebted to Professor Newall for assistance during the progress of the work and in the preparation of the paper, and to Mr. Manning for help in the preparation of the illustrations.

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*Note on Dr. Goldsbrough's Paper on "The Influence of Satellites upon the Form of Saturn's Ring."** By W. M. H. Greaves, B.A., Isaac Newton Student, University of Cambridge.

Dr. Goldsbrough's paper is an illustration of the care which must be exercised in superposing small effects. He considers the behaviour of a ring of small particles, revolving round a planet and subject to the gravitational influence of a satellite. Each particle, therefore, is subject to the perturbing influences of the other particles in the ring and also to the perturbing influence of the satellite. It is very natural to suppose that we could determine the effects of these perturbations separately and then superpose the results, but Dr. Goldsbrough's work indicates that if we do this our results may lead us to a view which is not only incorrect, but which gives us a completely wrong impression of the actual state of affairs.

First of all consider the behaviour of each particle of the ring as affected by the disturbing influences of the other particles, the effect of the satellite being disregarded for the moment. Maxwell's theory of the rings of Saturn indicates that provided the mass of the planet is large enough, the radius vector is expressible by the sum of a number of periodic terms of the form $A \cos(\alpha t + \beta)$, where t as usual denotes the time.

Now consider the behaviour of each particle as affected by the disturbing influence of the satellite alone. The Planetary Theory tells

* *Phil. Trans.*, A, 222, 101–130.

us that its radius vector is again expressible by the sum of a number of periodic terms of the form $A \cos(\alpha t + \beta)$, such a mode of expression being at any rate valid for a certain range of values of the time.

One is tempted to superpose these effects (which are each quite small), and to conclude that when we take into account the disturbing influences both of the other particles in the ring and also of the satellite, the expression for the radius vector of any particle consists of the sum of a number of periodic terms of the type $A \cos(\alpha t + \beta)$.

Dr. Goldsbrough shows that this view is incorrect and that the expression for the radius vector contains terms of the type $Ae^{ct} \cos(\alpha t + \beta)$.

Each exponent c is either purely real or purely imaginary. When it is purely imaginary (as is generally the case) e^{ct} becomes a periodic function of t with a real period, and again the radius vector is expressible as the sum of a number of periodic terms.

But Dr. Goldsbrough shows that there are certain regions in which some of the exponents c are real, so that the expression for the radius vector contains terms which represent periodic inequalities with steadily increasing amplitudes. In these zones the impression of stability which we would get by superposing the effects would not be correct.

The remarks concerning the radius vector of each particle are also applicable to the variation of its longitude from the mean value.

It should be noted that in general c is very small, so that for a very long time the term $Ae^{ct} \cos(\alpha t + \beta)$ is practically $A \cos(\alpha t + \beta)$. The tendency to instability is in fact very slow.

Dr. Goldsbrough has arrived at his results by considering the small oscillations of each particle about its position of relative equilibrium in the ring, and, as is usual in the mathematical treatment of such problems, he has considered a first approximation only. That is to say, he has neglected the squares and products of the small quantities ρ_λ and σ_λ which appear in his paper. Now there are certain reasons for *suspecting* that his zones of instability will disappear if we proceed to higher approximations, and that the impression obtained by considering the first approximation only is in fact illusory. These reasons will now be set forth.

Dr. Goldsbrough denotes the variations of the radius vector and longitude of any particle from its mean value by ρ and σ. He obtains linear differential equations with periodic coefficients for these quantities. The solution consists of a complementary function and a particular integral.

The complementary function consists of terms of the form $e^{c\phi}A$, where A is a periodic function of the independent variable ϕ, this being a certain linear function of the time t.

Dr. Goldsbrough seeks to determine the conditions which must be satisfied in order to ensure that some of the exponents c are purely real quantities. We find (section 4, equation (27) of his paper) that in the case when all the particles in the ring are of equal mass, the zones of values of a certain quantity κ for which this is the case are in the immediate neighbourhoods of those values of κ which satisfy the equation

$$(\Theta_{10} - n^2)(\Theta_{50} - n^2) - 4\kappa^2 n^2 = 0,$$

n being any integer. The quantity κ is a function of a the radius of the ring. In section 4, equation (30) of his paper, Dr. Goldsbrough points out that in the particular integral there appear terms with the denominators

$$(\Theta_{10} - m^2)(\Theta_{50} - m^2) - 4\kappa^2 m^2,$$

where m is any integer.

So that when the conditions are such that there appears a term with a vanishing denominator in the particular integral, there are terms in the complementary function involving real exponentials, and there is a zone of instability.

Similar results are true when the particles of the ring have unequal masses.

Now let us for a moment neglect the effect of the particles of the ring on each other and consider the behaviour of a single particle as influenced by the satellite alone. If we proceed according to the method of variation of arbitrary constants used in the Planetary Theory and take a first approximation with regard to $\frac{m'}{M}$, where m' is the mass of the satellite and M that of the planet, we obtain terms of the form

$$\frac{C}{pn - qn'} \cos\{(pn - qn')t + Q\},$$

where n and n' are the mean motions of the particle and the satellite respectively, p and q are integers, and the remaining quantities are constants.

If n and n' are nearly commensurable so that $pn = qn'$ nearly, the amplitude of the inequality will become very large. There is in fact an infinite value of the expression when the conditions are such that $pn - qn' = 0$.

Now it has been shown that if we take the terms arising from the higher approximations into account, the terms arising at various stages in the successive approximations may be grouped together in such a way that the vanishing denominator $pn - qn'$ is replaced by a denominator which does not vanish.* Clearly this term with a vanishing denominator corresponds to the term with a vanishing denominator in Dr. Goldsbrough's Particular Integral. We have seen that in the case of a particle perturbed by the satellite alone, the terms with vanishing denominators obtained by a first approximation are illusory and such terms do not appear when the subsequent approximations are taken into account. It seems then probable that if we proceed to higher approximations in the consideration of the behaviour of a particle when disturbed by other particles and also by the satellite, which is the problem treated by Dr. Goldsbrough, that the terms with vanishing denominators which appear in the first approximation would be replaced by terms which have no such vanishing denominators.

The following question then arises:—When we take into account the higher approximations, would the identity of the zones of instability in which we find exponential terms with real exponents in the com-

* Tisserand, *Méc. Céleste*, vol. iv., ch. xxv.

plementary function, and the zones in which there are vanishing denominators in the particular integral, an identity which Dr. Goldsbrough has shown to be true in the first approximation, still hold? If it does, then, as we have seen that the latter zone would probably disappear, it would follow that the former zones would also disappear, and that the conclusions as regards the existence of zones of instability, which Dr. Goldsbrough forms from the consideration of the first approximation only, are illusory.

There is at any rate a suspicion that this might be so. The problem then cannot be regarded as having been solved, and it cannot be so regarded until methods have been devised for taking into account the terms arising from the higher approximations. This would be a very laborious task and is probably beyond the scope of present-day mathematics.

In discussing matters of this kind we are apt to regard the validity of results obtained from a first approximation as an article of faith. But it must be remembered that the utmost we can say as regards such results is, that they give us an approximately correct picture of the physical phenomena involved for a very long time, and we must admit that the neglected terms may make their presence felt eventually. Now owing to the smallness of the exponents c, the instability obtained by Dr. Goldsbrough in a first approximation is of an extremely slow nature and will certainly take a very long time before its effect could be felt, so that it is quite possible that at this remote date the effect of the neglected terms might be appreciable, and there is no occasion for surprise if it should happen that they upset Dr. Goldsbrough's conclusions as regards the existence of zones of instability.

It should be noted that if this suspicion turns out to be correct we still have an explanation of the divisions in Saturn's rings to fall back upon. These divisions exist in those regions where the mean motions of particles would be nearly commensurable with the mean motion of one of Saturn's satellites, and in such regions, if we consider the perturbations due to the satellite alone, we find that there exist inequalities whose amplitudes, although they do not tend to become infinitely large, are large in comparison to the amplitudes of the inequalities of particles situated in other portions of the ring.* Consequently in such regions there is a greater tendency to collision and a smaller density, so that it is very probable that in the divisions of Saturn's rings particles do exist, but that the density of the matter in the divisions is much smaller than the density of matter in the rings themselves, and they appear to us as divisions in the rings.

Such an explanation would not only account for the Cassini Division, but also for the divisions in Ring B observed by Lowell, whose existence, be it noted, are not explained by Dr. Goldsbrough's theory.

* In general, if m' is the mass of the perturbing body, and M that of the central body, the amplitudes of the inequalities are of order m'/M. In the regions of close approach to commensurability of mean motions the amplitudes of the inequalities in radius vector are of order $\sqrt{m'/M}$, and the amplitudes of the inequalities in longitude are much larger.

On the Behaviour of a Small Body within the Cassini Division of Saturn's Ring. By W. M. H. Greaves, B.A., Isaac Newton Student, University of Cambridge.

Barnard's observations of Saturn's ring during the "disappearance" of 1907 (*M.N.R.A.S.*, **68**, 1908 March 5) led him to suspect that matter existed within the Cassini division. Owing to the fact that the mean Saturnicentric motion of a small body in such a position would be nearly double that of the satellite Mimas, some of the periodic perturbations due to the influence of Mimas would be considerable. The object of this investigation is to discover whether these perturbations would be large enough to carry the body into one or other of the bright rings during the course of its oscillation. If this were the case, it is very probable that it would sooner or later collide with the matter constituting the ring, and would be retained therein, so that it would be impossible for matter to exist permanently within the Cassini division.

The principal perturbations are two in number, and arise from the following two terms in the disturbing function :—

$$-\frac{m'}{2a'}\left(4b^{(2)}+a\frac{db^{(2)}}{da}\right)e\cos(2\mathrm{L}'-\mathrm{L}-\varpi) \quad . \quad . \quad . \quad (1)$$

$$\frac{m'}{2a'}\left(3b^{(1)}+a\frac{db^{(1)}}{da}-4a\right)e'\cos(2\mathrm{L}'-\mathrm{L}-\varpi') \quad . \quad . \quad (2)$$

With the usual notation, the unaccented symbols referring to the small body and the accented symbols to Mimas, L and L′ being the mean longitudes.

The mass of Saturn has been taken as the unit of mass. The other units are so adjusted that the numerical value of the constant of gravitation is unity. m' is the mass of Mimas.

It is assumed that all the motion is in the plane of the ring, and that the motion of Mimas is elliptic.

The inequalities in the motion of the small body arising from these two terms in the disturbing function will be denoted by inequalities I. and II. respectively.

First of all consider inequality I. Owing to the fact that e, the eccentricity of the small body, is small, it takes a very simple form.

Let r denote the radius vector of the small body. The expression for r obtained by taking a first approximation with regard to m' is given by Tisserand. We shall group together those terms in the expression for r which do not involve e and e'. Denoting this group by r_1 we have *

$$\frac{r_1}{a}=1+\tfrac{1}{2}m'\sum_{i=-\infty}^{i=+\infty}\mathrm{C}_i\cos i\left[(n-n')t+\epsilon-\epsilon'\right]$$

$$+m'\frac{a^2}{a'^2}\,\frac{\nu-3}{\nu(1-\nu)(2-\nu)}\cos\left[(n-n')t+\epsilon-\epsilon'\right],$$

* Tisserand, *Méc. Céleste*, vol. i., ch. xxii., formulæ (*b*), (*f*), and (*g*).

where

$$\nu = \frac{n'}{n},\ n \text{ and } n' \text{ being the mean motions,}$$

$$C_i = \frac{2i}{z_i(1-z_i)}\alpha A^{(i)} + \frac{1}{1-z_i}\alpha^2\frac{\partial A^{(i)}}{\partial \alpha},$$

$$z_i = i(1-\nu).$$

Now n is nearly equal to $2n'$, so that $1-z_2$ is small and C_2 is comparatively large.

Retaining the term involving C_2, and neglecting other terms in comparison, we have

$$\frac{r_1}{a} = 1 + e_1 \cos 2[(n-n')t + \epsilon - \epsilon'] \quad . \quad . \quad . \quad (3)$$

where

$$e_1 = \tfrac{1}{2}m'C_2.$$

The amplitude of the inequality in radius vector is ae_1. Expressing C_2 in partial fractions, and retaining only the principal part, we have

$$C_2 = \frac{1}{1-z_2}\left\{4\alpha A^{(2)} + \alpha^2\frac{dA^{(2)}}{d\alpha}\right\},$$

so that

$$C_2 = \frac{G_2}{1-z_2},$$

where

$$G_2 = \alpha\left(4b^{(2)} + \alpha\frac{db^{(2)}}{d\alpha}\right).$$

We can obtain good approximate results by calculating G_2 for $\alpha = 2^{-\frac{2}{3}}$, this being the value of $\alpha\left(=\frac{a}{a'}\right)$, for which $n = 2n'$ exactly.

This gives

$$G_2 = 1{\cdot}4999.$$

Using the relations $n^2a^3 = 1$, $n'^2a'^3 = 1+m$, we obtain $1 - z_2 = 2\alpha^{\frac{3}{2}} - 1$. Hence

$$C_2 = \frac{1{\cdot}4999}{2\alpha^{\frac{3}{2}} - 1},$$

and so

$$e_1 = \tfrac{1}{2}m' \times \frac{1{\cdot}4999}{2\alpha^{\frac{3}{2}} - 1} \quad . \quad . \quad . \quad . \quad (4)$$

Now take as the unit of length that distance which will subtend an angle of one second of arc when viewed from a distance of one astronomical unit.

Then the Saturnicentric distances of the inner and outer boundaries of the Cassini division and of its centre are 161·8, 167·0, and 164·4 respectively.

We shall assume that the mean motion n' of Mimas is exactly half the mean motion n of a particle situated at the centre of the division for which $a = 164{\cdot}4$.

Then the relations $n^2a^3 = 1$, $n'^2a'^3 = 1 + m'$ give

$$a' = 260{\cdot}97.$$

The actual value of a' for Mimas is 255·78. We have replaced Mimas by a fictitious satellite which would have the maximum perturbing effect within the division.

For the mass of Mimas we shall take

$$m' = \frac{1}{16340000}.$$

Calculating the values of e_1 as given by (4) for small bodies revolving at various mean distances a within the division, we obtain the following table :—

a.	e_1.
161·9	0·000,002020
162·4	0·000,002525
162·9	0·000,003360
163·4	0·000,00504
163·9	0·000,01013
164·4	∞
164·9	0·000,01007
165·4	0·000,00501
165·9	0·000,003345
166·4	0·000,002511
166·9	0·000,002013

Corresponding to $a = 164{\cdot}4$, e_1 is infinite. This result is, however, illusory. In the analysis we have retained only those terms in the expression for r which arise from the first approximation with respect to m', and for values of a near to 164·4 (when $a = 164{\cdot}4$, $n = 2n'$) the terms arising from the second and succeeding approximations become important and cannot be neglected. So the results do not apply to the region of values of a near $a = 164{\cdot}4$.

When the subsequent approximations are retained, the small vanishing term $n - 2n'$, which appears in the denominator, is replaced by a small non-vanishing divisor of the type *

$$\sqrt{(n - 2n')^2 + \sigma^2 \sin^2 \tfrac{1}{2}\theta},$$

so that the values of e_1 given in the table are too large. However, with the exception of the infinite value corresponding to $a = 164{\cdot}4$, they probably give a good approximation.

Bearing in mind that the tabulated values are too large, we now see that the amplitude ae_1 of the inequality is not large enough to ensure particles in the division being trapped in the rings. There remains a doubt as to the fate of particles whose mean distances lie between 163·9 and 164·9.

* Tisserand, *Méc. Céleste*, vol. iv., ch. xxv., p. 438.

We now consider inequality II., which arises from the term (2) in the disturbing function. In order to investigate this inequality we reduce the disturbing function to the term (2) only.

Write $L = \sqrt{a}$
$G = \sqrt{a(1-e^2)}$

l = mean anomaly of small body in its instantaneous ellipse.
$g = \varpi + \varpi' - 2L'$.

The equations of motion are

$$\left.\begin{aligned} \frac{dL}{dt} &= \frac{\partial F}{\partial l} & \frac{dl}{dt} &= -\frac{\partial F}{\partial L} \\ \frac{dG}{dt} &= \frac{\partial F}{\partial g} & \frac{dg}{dt} &= -\frac{\partial F}{\partial G} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (5)$$

where

$$F = \frac{1}{2a} + 2n'G + \frac{m'}{2a'}\left(3b^{(1)} + a\frac{db^{(1)}}{da} - 4a\right)e' \cos\theta,$$
$$\theta = l + g.$$

We write

$$F = A\cos\theta + B.$$

These equations have the integral

$$A\cos\theta + B = C \quad . \quad . \quad . \quad . \quad (6)$$

where C is a constant.

From (5) we have

$$\frac{dL}{dt} = -A\sin\theta, \qquad \frac{dG}{dt} = -A\sin\theta.$$

Hence

$$L - G = \text{a constant} \quad . \quad . \quad . \quad . \quad (7)$$

Furthermore, using (6), we get

$$dt = \pm \frac{dL}{\sqrt{A^2 - (C-B)^2}} \quad . \quad . \quad . \quad . \quad (8)$$

Let a_1, e_1, etc., denote the initial values of a, e, etc.

Write

$$a = a_1(1+x) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Neglecting terms of order $m'x$ we have $A = A_1$.

Again from (6),

$$C - B = A_1\cos\theta_1 + B_1 - B,$$

and we easily obtain the expansion

$$B - B_1 = x\left\{n'L_1 - \frac{1}{2a_1}\right\} + x^2\left\{\frac{1}{2a_1} - \tfrac{1}{4}n'L_1\right\} + o(x^3).$$

Put

$$\left.\begin{aligned} D_1 &= \frac{2n'}{n_1} - 1 \\ D_2 &= 1 - \tfrac{1}{2}\frac{n'}{n_1} \end{aligned}\right\}$$

since $n_1 = 2n'$ nearly, D_1 is small and $D_2 = \frac{3}{4}$ approx.

Using the relation $n_1^2 a_1^3 = 1$, we obtain

$$B - B_1 = \frac{1}{2a_1}\{D_1 x + D_2 x^2\},$$

terms of order x^3 being neglected.

Hence

$$B - C = \frac{1}{2a_1}\{D_1 x + D_2 x^2 - 2a_1 A_1 \cos\theta_1\},$$

so that

$$A + B - C = \frac{1}{2a_1}\{W \sin^2 \tfrac{1}{2}\theta_1 + D_1 x + D_2 x^2\} = \frac{U_1}{2a_1} \text{ say},$$

$$A - B + C = \frac{1}{2a_1}\{W \cos^2 \tfrac{1}{2}\theta_1 - D_1 x - D_2 x^2\} = \frac{U^2}{2a_1} \text{ say},$$

where

$$W = 4a_1 A_1 = 2m'a_1\left(3b^{(1)} + a_1\frac{db^{(1)}}{da_1} - 4a_1\right)e' \quad . \quad . \quad (10)$$

For the range of values of a considered, W is positive.

We now get

$$A^2 - (B - C)^2 = \frac{U_1 U_2}{4a_1^2},$$

and (8) becomes

$$n_1 dt = \pm \frac{dx}{\sqrt{U_1 U_2}},$$

terms of order x^3, of order $m'x$, and of order $D_1 x^2$ being neglected.

Write

$$y = x + \frac{D_1}{2D_2}.$$

We obtain

$$n_1 D_2 dt = \pm \frac{dy}{\sqrt{\left\{y^2 + \frac{4D_2 W \sin^2 \frac{1}{2}\theta_1 - D_1^2}{4D_2^2}\right\}\left\{\frac{4D_2 W \cos^2 \frac{1}{2}\theta_1 + D_1^2}{4D_2^2} - y^2\right\}}} \quad (11)$$

We now have two cases to consider.

Case A.

Suppose

$$D_1^2 > 4D_2 W \sin^2 \tfrac{1}{2}\theta_1 \quad . \quad . \quad . \quad . \quad (12)$$

Write

$$p^2 = \frac{D_1^2 - 4D_2 W \sin^2 \frac{1}{2}\theta_1}{4D_2^2}$$

$$p'^2 = \frac{D_1^2 + 4D_2 W \cos^2 \frac{1}{2}\theta_1}{4D_2^2}.$$

Then we get

$$n_1 D_2 dt = \pm \frac{dy}{\sqrt{(y^2 - p^2)(p'^2 - y^2)}}.$$

This gives on integration,

$$y = \pm p' \mathrm{dn} u, \text{ so that } x = \pm p' \mathrm{dn} u - \frac{D_1}{2D_2},$$

where $u = n_1 D_2 (t+c)$, c being a constant of integration, and k, the modulus of elliptic functions, is given by

$$k^2 = 1 - \frac{p^2}{p'^2} = \frac{W}{p'^2 D_2}.$$

Let R_x and R_α denote the ranges of variation of x and α.
Then

$$R_x = p' - p, \qquad R_\alpha = \alpha_1 (p' - p).$$

Now, α_1 being fixed in advance, let M denote the maximum values of R_α for different values of θ_1.

We easily find

$$p' - p = \frac{1}{2D_2}\{\sqrt{4D_2 W + z^2} - z\},$$

where

$$z^2 = D_1^2 - 4D_2 W \sin^2 \tfrac{1}{2}\theta_1.$$

(*a*) If $D_1^2 > 4D_2 W$, θ_1 may take all values; and z may take all values from z_1 to $+\infty$ where $z_1 = \sqrt{D_1^2 - 4D_2 W}$.

The maximum value of R_α occurs when $z = z_1$, and we have

$$M = \frac{\alpha_1}{2D_2}(D_1 - \sqrt{D_1^2 - 4D_2 W}).$$

(*b*) If $D_1^2 < 4D_2 W$, the values of θ_1, which are admissible in order that (12) may be satisfied, are restricted, z may vary from 0 to $+\infty$; and the maximum value of R_α occurs when $z = 0$, giving

$$M = \alpha_1 \sqrt{\frac{W}{D_2}}.$$

Case B.

Suppose

$$D_1^2 < 4D_2 W \sin^2 \tfrac{1}{2}\theta_1.$$

Write

$$p''^2 = \frac{4D_2 W \sin^2 \tfrac{1}{2}\theta_1 - D_1^2}{4D_2^2}$$

$$p'^2 = \frac{D_1^2 + 4D_2 W \cos^2 \tfrac{1}{2}\theta_1}{4D_2^2}.$$

Then we get

$$n_1 D_2 dt = \pm \frac{dy}{\sqrt{(y^2 + p''^2)(p'^2 - y^2)}},$$

giving

$$y = \pm p' \operatorname{cn} v, \text{ so that } x = \pm p' \operatorname{cn} v - \frac{D_1}{2D_2},$$

where

$$v = n_1 D_2 \sqrt{(p'^2 + p''^2)}(t+c),$$

the modulus k of elliptic functions being given by

$$k^2 = \frac{p'^2}{p'^2 + p''^2}.$$

We now have

$$R_x = 2p'$$
$$R_a = 2a_1 p'.$$

a_1 being fixed, R_a is a maximum for different values of θ, when $\cos^2 \frac{1}{2}\theta_1$ is a maximum.

Now we are restricted to values of θ, given by $\sin^2 \frac{1}{2}\theta_1 > \frac{D_1^2}{4D_2W}$.

Hence $\cos^2 \frac{1}{2}\theta_1$ is a maximum when

$$\sin^2 \tfrac{1}{2}\theta_1 = \frac{D_1^2}{4D_2W}$$

and

$$M = 2a_1\sqrt{\frac{W}{D_2}}.$$

Combining the results of the two cases A and B, we see that when $D_1^2 > 4D_2W$ we have

$$M = \frac{a_1}{2D_2}\{D_1 - \sqrt{D_1^2 - 4D_2W}\},$$

and when $D_1^2 < 4D_2W$ we have

$$M = 2a_1\sqrt{\frac{W}{D_2}},$$

M being the maximum value of R_a for a given initial value a_1 of a.

These formulæ enable us to calculate M for various values of a_1 corresponding to various positions of a small body within the Cassini division. We shall obtain very good approximate results by evaluating W for the value $\frac{a}{a'} = \alpha = \frac{1}{2\frac{2}{3}}$, corresponding to $a = 164{\cdot}4$, the centre of the division.

With this value of α, taking $e' = 0{\cdot}0190$ and $m' = \frac{1}{16340000}$, we obtain $\log W = \bar{9}{\cdot}10$.

The values of M for different values of a are now easily computed. The results are as follows:—

a_1.	M.
161·9	∓0·0000089
162·4	0·0000112
162·9	0·000015
163·4	0·000022
163·9	0·000045
164·4	0·014
164·9	0·000046
165·4	0·000023
165·9	0·000015
166·4	0·0000114
166·9	0·0000091

This table shows that the ranges of variation of a are not large enough to take the small body out of the division. This result remains true when we take into account the possible values of the eccentricity e. Using (7) we have, on expanding in powers of x,

$$e^2 = e_1^2 + x\sqrt{1-e_1^2}(\sqrt{1-e_1^2} - \tfrac{1}{2}) + o(x^2),$$

so that if e is small initially it remains small.

We conclude that the perturbations due to Mimas are not large enough to ensure a small body initially in the Cassini division being trapped in one of the bright rings.

The foregoing theory takes into account only the principal perturbations due to Mimas and neglects those due to the other satellites and to the particles of the rings and other particles within the division. Dr. Goldsbrough (*Phil. Trans.*, A, 222, 101–130, Oct. 1921) has considered the case of a ring of small particles revolving round Saturn and disturbed by a satellite, the mutual attractions of the particles being taken into account. Dr. Goldsbrough takes a first approximation and obtains linear differential equations for the radius vector and longitude of each particle of the ring. The solution consists of a complementary function and a particular integral, and terms of the type Ae^{ct}, where A is a periodic function of t, appear in the complementary function. There are certain regions in which the exponents c are real, indicating that the ring is unstable. These regions constitute areas of clearance for the satellite in question. For rings within these regions there also appear terms with small denominators in the particular integrals. The Cassini division is found to correspond to one of the regions of clearance of the satellite Mimas.

Owing to the smallness of the coefficients c the instability of rings within the areas of clearance develops slowly, and it is by no means apparent that the process of dissipation has been completed. Indeed it is certain that in one case at least it has not, for one of the areas of clearance corresponding to the satellite Titan includes the whole system of rings.

It is not certain whether Dr. Goldsbrough's conclusions as regards the existence of zones of instability remain correct when the terms arising from higher approximations are taken into account, but in any case, owing to the slow development of the instability, there is no apparent reason why matter should not exist within the Cassini division provided the amplitudes of the perturbations represented by the terms with small denominators in the particular integrals are not so large as to ensure particles initially in the division being trapped in one of the bright rings. Now these perturbations are practically the perturbations due to Mimas that we have been considering,* and we have found that they are not large enough for this to take place. It would thus appear that there is no objection to Barnard's hypothesis that matter is to be found within the Cassini division.

* Strictly speaking, this is only true when any process of clearance that may exist has already proceeded to such an extent that the density of the matter remaining in the division is comparatively small.

The Temperature in the Outer Atmosphere of a Star.

By E. A. Milne, M.A.

(*Communicated by the Director, Solar Physics Observatory, Cambridge.*)

§ 1. It was first shown by Schwarzschild * that when a mass of material is in radiative equilibrium the temperature distribution is such that the boundary assumes a definite non-zero temperature which depends on the total radiation emitted through the boundary. Schwarzschild's precise result is as follows. Consider a small portion of the boundary which may be regarded as plane, and suppose the material immediately inside is stratified in planes parallel to the boundary; then, provided the material is *gray* (*i.e.* has an absorption coefficient which is the same for all wave-lengths), the boundary will possess a temperature T_0 given approximately by $T_0^4 = \frac{1}{2}T_1^4$, where T_1 is the effective temperature of the surface as deduced from Stefan's law. The same formula was obtained, though less generally, by Humphreys.† Humphreys' more physical line of argument makes it immediately clear why the coefficient of T_1^4 in the formula is approximately $\frac{1}{2}$, but detailed investigations have shown that the true value lies between $\frac{1}{2}$ and $\frac{3}{8}$, and that a better value is $1/2{\cdot}3 = 0{\cdot}435$. However, in the following we shall for simplicity adopt the value $\frac{1}{2}$.

§ 2. The object of this note is to consider the effect of the removal of the restriction that the material must be gray. Schwarzschild's formula is not applicable to material which absorbs and emits selectively.

The essence of the matter can be quickly gathered from two special cases. Let $B_\lambda(T)$ be the intensity of black radiation at temperature T and wave-length λ. Then, T_0 and T_1 having the same meaning as before and the outflowing radiation being taken to be approximately black, the crude condition of radiative equilibrium at the boundary gives

$$2\int_0^\infty k_\lambda B_\lambda(T_0)d\lambda = \int_0^\infty k_\lambda B_\lambda(T_1)d\lambda \quad . \quad . \quad . \quad (1)$$

where k_λ is the absorption coefficient, in λ, of the material near the boundary. (This may easily be seen by Humphreys' method.) Now take the special case in which k_λ is zero save in the extreme infra-red (say beyond λ'), where its value is taken to be constant and equal to k. In this part of the spectrum we may adopt the Rayleigh-Jeans approximation for $B_\lambda(T)$, and we have

$$2\int_{\lambda'}^\infty 2cRT_0\lambda^{-4}kd\lambda = \int_{\lambda'}^\infty 2cRT_1\lambda^{-4}kd\lambda,$$

which gives

$$T_0 = \tfrac{1}{2}T_1.$$

Again, take the special case in which k_λ is zero save in the ultra-violet (say beyond λ''), where its value is taken to be constant and equal to k.

* *Gott. Nach.*, 1906, p. 41.
† *Astrophys. Journ.*, 29, 14 (1909).

In this part of the spectrum we may adopt Wien's approximation, and we find

$$2\int_0^{\lambda''} k\lambda^{-5}e^{-hc/\lambda RT_0}d\lambda = \int_0^{\lambda''} k\lambda^{-5}e^{-hc/\lambda RT_1}d\lambda.$$

Putting $hc/\lambda RT_0 = x$, $hc/\lambda RT_1 = y$, we find

$$2T_0^4\int_{x''} e^{-x}x^3dx = T_1^4\int_{y''}^{\infty} e^{-y}y^3dy.$$

But when X is large we have the asymptotic formula

$$\int_X^{\infty} e^{-x}x^3dx \sim e^{-X}X^3.$$

Hence, since by hypothesis x'' and y'' are large,

$$2T_0e^{-x''} = T_1e^{-y''}.$$

Putting now

$$T_1/T_0 = x''/y'' = \mu,$$

this becomes

$$\tfrac{1}{2}\mu = e^{y''(1-\mu)}$$

or

$$y'' = \frac{\log 2 - \log \mu}{\mu - 1},$$

whence it appears that when y'' is very large μ is very close to unity. Thus for λ'' sufficiently small T_0 will approach as close as we please to T_1.

§ 3. Consideration of these two extreme cases suggests that *in all cases we shall have*

$$T_1 > T_0 > \tfrac{1}{2}T_1 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We shall now prove this rigorously. Substituting Planck's formula in (1), we have

$$\int_0^{\infty} \frac{2e^y - e^x - 1}{(e^x - 1)(e^y - 1)} k_\lambda \lambda^{-5}d\lambda = 0 \quad . \quad . \quad . \quad (3)$$

Now this equation cannot be satisfied if the expression

$$2e^y - e^x - 1$$

is always positive. Consequently, there must exist at least one value of λ for which

$$e^x > 2e^y - 1,$$

i.e. for which

$$x > y + \log(2 - e^{-y}).$$

But $x/y = T_1/T_0$. Consequently, there must exist at least one value of λ for which

$$1 + \frac{\log(2 - e^{-y})}{y} < \frac{T_1}{T_0}.$$

Calling the expression on the left of this inequality $f(y)$, we see that T_1/T_0 must exceed the least value of $f(y)$ for values of $y>0$. The lower bound of $f(y)$ is 1, which is approached as $y\to\infty$. Hence

$$\frac{T_1}{T_0}>1 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But precisely similar arguments show that T_1/T_0 cannot exceed the greatest value of $f(y)$. The upper bound of $f(y)$ is 2, which is approached as $y\to 0$. Hence

$$\frac{T_1}{T_0}<2 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Inequalities (4) and (5) constitute the result (2). As a corollary we have that the boundary temperature will be greater or less than the Schwarzschild value according as the boundary material is more absorbent on the blue or red side of the maximum.*

§ 4. We conclude with an extension of these results to the temperature gradients in that portion of a star's atmosphere which lies outside the apparent limb. The Schwarzschild boundary temperature is deduced on the assumption that the radiation incident on the boundary from the outside is zero, and it is legitimately applied to give the temperature at the limb, since at the limb the back radiation from the outlying matter is negligible compared with the outward radiation. (That this is so follows almost from the definition of "limb"; for along a line of sight which passes just outside the limb the integrated optical mass is almost zero, radially the optical mass outside the limb must be smaller still, and so the back radiation (which is proportional to the optical mass) must be negligible.) Now outside the limb at higher levels the back radiation will be smaller still, and consequently the considerations which lead to the Schwarzschild boundary temperature are applicable in determining the temperature of radiative equilibrium at these higher levels from the net outward flux per unit area. At the limb this is measured as the average radiation per unit area of the apparent disc, but outside the limb the net flux falls off as the inverse square of the distance from the centre. Consequently, for gray material outside the limb T^4 falls off as the inverse square, and the temperature T as the inverse square root, of the distance from the centre of the star. The formula

$$T_r^4 = T_0^4\left(\frac{r_0}{r}\right)^2 = \tfrac{1}{2}T_1^4\left(\frac{r_0}{r}\right)^2$$

gives, in fact, the temperature assumed by a slab of gray material placed normal to the radiation at any distance from the star whatever. In

* This result must be distinguished from that of *M.N.*, **81**, 510 (1921), where it was shown that the presence of a strong absorption line would *lower* the boundary temperature. But in that investigation the layers contributing the absorption line were supposed to extend to a considerable depth, so that the outward flowing radiation near the boundary was no longer black, being already largely deprived of the wave-lengths most strongly absorbable by the boundary material. The present case is that of a *very thin* selectively absorbing layer at the boundary.

particular, it is known to give the order of magnitude of the temperature of the earth * (assumed to be a gray body at a given distance from the sun), when a correction is made for its spherical form.

We consider now the modification of this law when the material is not gray. Instead of equation (1) we have

$$2\int_0^\infty k_\lambda B_\lambda(T_r)d\lambda = \left(\frac{r_0}{r}\right)^2 \int_0^\infty k_\lambda B_\lambda(T_1)d\lambda.$$

A train of reasoning similar to that used in § 3 shows that T_1/T_r must lie between the greatest and least values of the function

$$f_r(y) = 1 + \frac{\log\left[2(r/r_0)^2(1-e^{-y})+e^{-y}\right]}{y},$$

for $y>0$. These are respectively $2(r/r_0)^2$ and 1, which occur as $y\to 0$ and as $y\to\infty$. It follows that

$$T_1 > T_r > \tfrac{1}{2}T_1\left(\frac{r_0}{r}\right)^2.$$

If, by analogy with the earth's atmosphere, we use the word *stratosphere* to denote the outer quiescent forms of a star's atmosphere, we may state the results as follows:—*If the stratosphere of a star is in radiative equilibrium, its temperature will fall off as the inverse square root of the distance provided it is composed of gray material. In any case, its temperature cannot fall off more rapidly than the inverse square of the distance, but the temperature distribution may be nearly isothermal, the exact law depending on the part of the spectrum in which it absorbs. The inverse square distribution corresponds to absorption in the far infra-red, the isothermal distribution to absorption in the far ultra-violet.* The fact that matter at any distance *may* take up a temperature indefinitely near the effective temperature of the central radiating body, provided the absorption lies sufficiently far in the ultra-violet, was pointed out by Fabry.†

These results are of use in discussing the escape of molecules from an atmosphere by diffusion.

Solar Physics Observatory, Cambridge;
1922 *March* 28.

* *E.g.* Poynting, "Radiation in the Solar System," *Phil. Trans.*, 202, A, 525 (1903).

† "The Temperature of Space," *Astrophys. Journ.* 45, 269 (1917).

The Spectra of Physically Connected Pairs and the Giant and Dwarf Theory. By Peter Doig.

(*Communicated by L. J. Comrie.*)

The components of binary systems, or of pairs known by common proper motion to be physically connected, may be fairly regarded as generally of coeval origin and of similar composition. The disparity in their spectra should thus constitute them "exceptionally trustworthy guides to the unravelment of evolutionary time-relations."* The study of these spectral differences may be expected, therefore, to throw some light on the two questions of individual stellar development and the birth of double-star systems themselves.† If, as has been assumed above, the origin of connected pairs has been by fissure of a larger star, or by condensation about two comparatively close but independent nuclei, some orderly relationship between differences in brightness and differences in spectra might reasonably be anticipated; on the other hand, a general or frequent origin by "capture" would be less likely to favour any definite connection between these features unless simultaneous origin can be attributed to the stars occupying very great volume of space.

Writing in 1917,‡ Aitken stated that when difference in spectral class had been accurately measured the fainter component appeared to be "as likely to belong to the later spectral class as the brighter one." He also gave a table of the particulars of nine pairs of which the companion showed a "later" spectrum than the primary in seven cases. In 1918 Perrine§ advanced highly speculative conclusions to account for the observed spectral differences of connected pairs, adopting a hypothesis of preferential distribution of "finely divided solid matter" in space which has not been supported by any independent evidence. Lau‖ and Leonard¶ have connected these differences with the Giant and Dwarf theory of stellar development, and it is on such lines that the present attempt to investigate the subject has been made.

By the theory of Russell and Hertzsprung,** the more quickly developing component of a pair of neighbouring stars of simultaneous origin would be expected to show generally:—

(*a*) in the Giant diffuse stage, when density and effective temperature are increasing, a so-called "earlier" type spectrum than that of the more slowly developing star;

(*b*) in the Dwarf stage, when density is increasing and temperature is falling, a so-called "later" type spectrum than that of the more slowly developing star.

* Miss A. M. Clerke, *Problems in Astrophysics*, p. 275 (1903).
† On theories of origin, see Aitken, *Binary Stars*, chap. xi. (1918).
‡ Aitken, *loc. cit.*, p. 264.
§ *Astrophysical Journal*, **47**, 5 (1918).
‖ *Astronomische Nachrichten*, **205**, 29 (1917).
¶ *Pub. Ast. Soc. Pacific*, **32**, 213 (1921).
** *Journal British Astronomical Association*, **32**, 2, 58; also *ibid.*, **32**, 1, 6–20 (1921–22).

At present the opinion which seems to be most widely held regarding stellar development is that the rate of evolution is generally inversely proportional to mass. With regard to the relation between brightness and mass, it has been shown on theoretical grounds, with reasonable assumptions, that luminosities are probably greater with mass for any particular stage of development as indicated by spectral type.* Since total luminosity does not appear to vary greatly during the Giant stage of any given star, owing to the compensatory effect between increased surface brightness and reduced superfices, the fainter of the two in a pair of Giants is very probably the less massive. Support is given to this by the fact that in spectroscopic binaries (which are for the most part Giants) for which orbits are known, "the brighter star is, with very few exceptions, the more massive."† In the case of Dwarf systems, those visual binaries for which relative masses have been estimated mostly fall into this division, and seem to have the fainter component also the smaller in mass, although there is as yet some discordance in the results.‡ It would, therefore, appear likely that the fainter and less massive stars in connected pairs will prove to be the more advanced in development.§

As preliminary evidence bearing on spectral differences, the following statement by W. W. Campbell,‖ referring to the spectroscopic binaries (chiefly Giant systems ¶), may be quoted. "It is fairly well established that when the two spectra are substantially equal in brightness they are identical in class; and when one spectrum is considerably fainter than the other, the spectrum of the secondary is apparently of a slightly earlier class than the spectrum of the primary. There appear to be no exceptions to this rule, though the difficulty in the way of giving accurate descriptions of the fainter spectrum must be recognised."

On the other hand, the fainter star in eclipsing variables "has always been found to be redder, and hence probably of a later class of spectrum than the bright star, whenever it has been possible to determine the relative colour-index of the two components."** These fainter stars are very often large and of small density, and are probably, in many cases, Giant companions of Giant primaries. There appears some discordance here from what is shown by the Giant non-eclipsing spectroscopic pairs. It is worthy of note, however, that when a pair of stars of not very dissimilar brightness are both of "early" type spectra, and therefore well advanced towards the turning point between the Giant and Dwarf stages, the fainter component may have *passed* this point and consequently have a spectrum similar to or even somewhat "later" than that of the primary.

An attempt has been made to obtain sufficient data on visual pairs

* *Observatory*, 40, 251 (1917).

† Aitken, *loc. cit.*, p. 208.

‡ Aitken, *loc. cit.*, p. 216; Campbell, *Stellar Motions*, pp. 274, 276, and Table XXX.

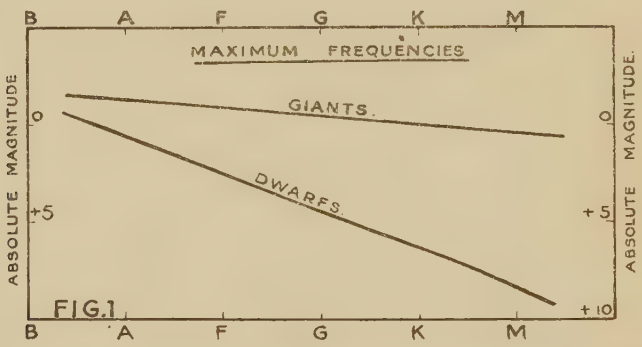
§ See Leonard, *loc. cit.*, p. 214, par. (4).

‖ Campbell, *loc. cit.*, p. 273.

¶ Shapley (*Mount Wilson Contributions*, No. 156, 2, 1918) gives the mean parallax of 90 eclipsing variables as 0·0013, which means high luminosity.

** Aitken, *loc. cit.*, p. 189.

for the investigation, and the magnitudes, individual spectra, and parallaxes (*giving distances which are only relied upon as sufficiently accurate for Giant and Dwarf classification*) have been collected from various sources* for 36 connected pairs in which the brighter star is apparently a Giant, the fainter being either a Giant or a Dwarf; and for 76 pairs in which all stars are probably Dwarfs. These are given in Tables I. and II. respectively. The criterion of the separation into Giants and Dwarfs is provided by a diagram of maximum frequencies of absolute magnitudes for each spectral type (fig. 1) based on the luminosities of 1749 stars, including 1646 of the list published by Adams and 103 from other sources.† This diagram has been derived from curves of the percentages of stars of different grades of absolute magnitude in each spectral class, the "peaks" of these curves providing the values of maximum frequency from which the lines for Giants and



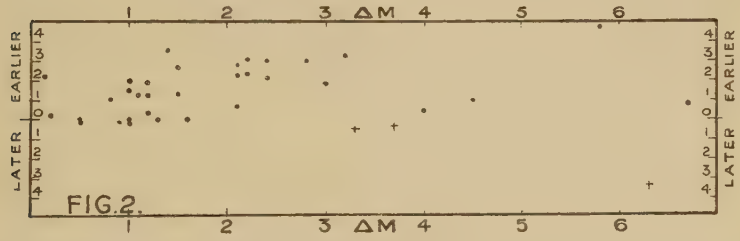
Dwarfs in fig. 1 are obtained. Figs. 2 and 3 show the data of the tables, being a graphic presentation of the difference in magnitudes (ΔM) and in spectral types (ΔT) for Tables I. and II. respectively. The column headings are: *m*, visual magnitudes; M, absolute magnitudes; T, spectral types; the suffixes *a* and *b* referring to the brighter and fainter components respectively. The terms "earlier" and "later" are used in their conventional meaning, but the writer is no doubt not alone in wishing that better expressions could be found.

In fig. 2, out of the 36 systems plotted, 3 are distinguished by special mark, being at first glance inconsistent with the others. They are α Leonis, α Crucis and Lac. 5147, and η Tauri. As the companions, being fainter and probably less massive than the primaries, may be reasonably expected to develop more quickly as a rule, the accordance with the expectation in (*a*) above is striking, and the three just mentioned can also be shown, it is believed, to be equally in agreement. Taking α Leonis, the case of apparent greatest nonconformity, the explanation would appear to be that the fainter star has in

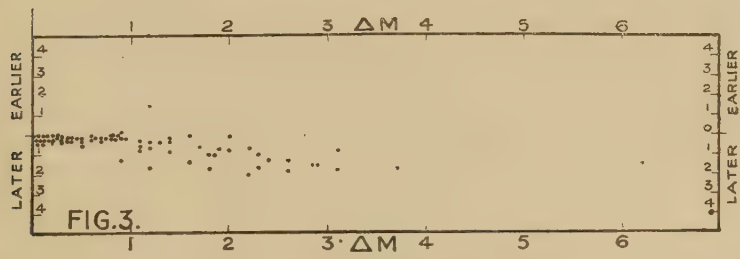
* Chiefly *H.C.O. Annals*, 56 (1912), *Mount Wilson Contributions*, No. 199 (1921), and *Greenwich Catalogue of Double Stars* (1893-1919). Professor W. S. Adams has also kindly provided the unpublished information for 11 pairs.

† Adams and others, *M. W. Cont.*, No. 199; see also *Journal B.A.A.*, 32, 2, 59.

all probability preceded its principal in development through the Giant stage, reached its maximum temperature (which may not have been higher than that corresponding to a comparatively "late" type), and then passed "backwards" as a cooling Dwarf, crossing through the spectral type of its Giant primary, and reaching K2. The star Lac. 5147 and the companions to η Tauri, and even those in such systems



as ϵ Hydræ, δ Boötis, α Ursæ Minoris, and others, appear to have similarly passed, returned close to, or to be passing, the spectral type of their Giant principals, which are probably generally on the upward temperature gradient. There are two Giant systems OΣ 517 (β.G.C. 2588) and OΣ 309 (β.G.C. 7587) in which the slight differences in magnitude (0·2) and the very small separations (less than half a second of arc) render very uncertain the identification of the objective-prism spectra. They are given* as having the fainter



star considerably "later" in type, and are mentioned here without further reference in the table or diagram. There are also other feasible explanations, such as that the primary in each case might happen to be a spectroscopic binary, when the constituent stars would very probably be each fainter than the visual companion, and thus reverse the connection between ΔT and ΔM.

Referring to fig. 3, in which the data for 75 systems are plotted, it will be seen that on the assumption that the fainter and less massive stars develop more quickly, the expectation of paragraph (*b*) above is fully realised. Only one of the pairs seems noticeably discordant, and in this case, χ Aquilæ, the parallax (a spectroscopic one),

* *Greenwich Catalogue of Double Stars*, 1893–1919, pp. xiii and xvi.

may be too large, and the system really belong to the group in Table I. On the other hand, as the separation is very small (0″·45) there may be inaccurate identification of the objective-prism spectra.

We have in Sirius and in o^2 Eridani double systems which are clearly out of the run of the ordinary relationships shown in Table II. and fig. 3. There may be some explanation which would show them to be, after all, normal, but apparently abnormal through some factor extraneous to ordinary laws of development. The writer has elsewhere * advanced the conjecture that the unusual faintness of these A type comites may be due to the light-scattering effect of surrounding dust clouds, and some slight support is given to the idea by the fact that the Lick observers † see the comes of Sirius "decidedly yellower" than the primary, which is of similar spectrum.

In an earlier paragraph capture has been referred to as one of the possible modes of origin of connected pairs. The chief objection usually advanced has been that close approaches could not have taken place in sufficient numbers to account for the large proportion of the stars which are double. The generally consistent relationships between differences of spectra and of brightness shown in the 111 pairs here considered, although only for a relatively small number of systems, would appear to strengthen objections to this theory.

If the results of this investigation can be taken as conclusive, a criterion of distance in connected pairs, and even in connected groups of stars, is provided. By means of such a diagram of maximum frequencies as fig. 1, an estimate of the most probable real luminosities, and consequently parallaxes, of connected systems for which the differences in spectra are known (and hence the Giant and Dwarf classification), can be made. This has been done in the accompanying Table III. for a number of pairs ‡ for which parallaxes are not known to the writer. A fairly consistent value of parallax for the cluster Præsepe has also been obtained from data in which two triple systems and one pair are concerned. For this cluster Van Rhijn, § by a method based on the assumption that the luminosity law is the same as that for the solar neighbourhood, has deduced a parallax of 0″·024, which, however, would give abnormally small absolute magnitudes for the A type stars here dealt with. On the other hand, Shapley || has derived an approximate parallax of 0″·003 "from measured apparent magnitudes combined with absolute magnitudes estimated on the basis of observed spectra," but it does not appear that any criterion was employed as to the classification of the stars into Giants and Dwarfs. By various methods, Schwarzschild found 0″·0063, Köhlschutter, 0″·0072, and Rasmuson, 0″·011.¶ The mean of all these is 0″·0103, with which the value in Table III. closely agrees.

* *Journal B.A.A.*, **32**, 2, 65 (1921–22).

† Aitken, *loc. cit.*, p. 230.

‡ *H.C.O. Annals*, **56** (1912).

§ *Observatory*, **42**, article by Schouten, "The Parallax of some Stellar Clusters" (1919).

|| *Proc. Nat. Acad. Sciences*, **5**, 60 (1919).

¶ *Meddelanden från Lunds Astronomiska Observatorium*, Nr. 26, p. 56 (1921).

Summary.

The results of this investigation may be set out as below:—

1. In connected pairs, in which the primary at least is of the higher grade of luminosity (a Giant), the fainter star is generally of a so-called "earlier" type of spectrum, and progressively so with increase in difference of brightness of components.

2. In connected pairs, both stars of which are of the lower grade of luminosity (Dwarfs), the fainter star is normally of a so-called "later" spectral type, and progressively so with increase in difference of brightness of components.

3. A criterion of Giant and Dwarf classification is thus provided, which it may be possible to extend statistically to clusters or groups of connected stars.

4. The fainter stars in connected systems seem generally further advanced in development than the brighter. This may be due to smaller mass and consequent quicker development.

Subsidiary conclusions are that:—

1. The capture theory of origin of physically connected pairs is unlikely to be a true one, unless in the improbable event of simultaneous origin of the stars throughout great volume of space.

2. More or less close approximations to parallax may be obtained when differences in brightness or spectra of constituent stars are considerable, by classification into Giants and Dwarfs, and the use of the most probable absolute magnitudes corresponding to the particular spectral types. The degree of accuracy of such estimates would appear to be greater in the case of Dwarfs, owing to smaller dispersion of luminosity for a given spectral type, than in the case of Giants.*

In conclusion, the author would like to remark that the foregoing paper was written practically as it stands before the abstracts of Leonard's results in *Publications of the Astronomical Society of the Pacific*, August 1921, came into his hands.

TABLE I.

Systems containing Giant Stars.

Star.	ma.	mb.	π.	Ma.	Mb.	Ta.	Tb.	ΔM.	ΔT.
			"						
α Urs. Min.	2·1	8·8	0·010	−2·9	3·8	F8	Fo	6·7	0·8 *e*
α Leonis	1·3	7·6	0·033	−1·1	5·2	B8	K2	6·3	3·4 *l*
α Scorpii	1·2	7·0	0·010	−3·8	2·0	Ma	B3	5·8	4·7 *e*
δ Boötis	3·5	8·0	0·028	0·7	5·2	Ko	Go	4·5	1·0 *e*
ϵ Hydræ	3·5 A	7·5 C	0·017	−0·4	3·6	F9	F5	4·0	0·4 *e*
α Crucis and Lac. 5147	1·5	5·2	0·010	−3·5	0·2	B1	B5	3·7	0·4 *l*

* See diagrams, Eddington, *Stellar Movements*, p. 171 (1914), and *Journal B.A.A.*, 32, 1, 18 (1921–22).

TABLE I.—*continued.*

Star.	ma.	mb.	π.	Ma.	Mb.	Ta.	Tb.	ΔM.	ΔT.
			″						
η Tauri	3·0	6·3	0·013	−1·4	1·9	B5	Ao	3·3	0·5 *l*
o^1 Cygni	3·9	7·1	0·006 S	−2·1	1·1	Ko	B8	3·2	3·2 *e*
β^1, β^2 Capricorni	3·2	6·2	0·007	−2·6	0·4	F8	Ao	3·0	1·8 *e*
γ Androm.	2·3	5·1 BC	0·010	−2·7	1·0	Ko	Ao	2·8	3·0 *e*
ϵ^1 ϵ^2 Boötis	2·7	5·1	0·015	−1·4	1·0	Ko	Ao	2·4	3·0 *e*
α Herculis	3·6	6·0	0·012 S	−1·0	1·4	Mb	F9	2·4	2·1 *e*
β Cygni	3·2	5·4	0·010	−1·8	0·4	Ko	B9	2·2	3·1 *e*
ι Cancri	4·2	6·6	0·021	0·8	3·0	G5	A2	2·2	2·3 *e*
Boss 4898	5·7	7·8	0·004 S	−1·4	−0·7	G8	Ao	2·1	2·8 *e*
β.G.C. 8114	6·5	8·6	0·006 S	0·4	2·5	K2	Fo	2·1	2·2 *e*
β.G.C. 11936	6·2	8·3	0·006	0·1	2·2	K4	G8	2·1	0·6 *e*
15 Aquilæ	5·5	7·1	0·005 S	0·8	2·4	K	K	1·6	0·0
24 Com. Ber.	5·2	6·7	0·011 S	0·3	1·8	Ko	A3	1·5	2·7 *e*
β.G.C. 12704	6·0	7·5	0·010 S	1·0	2·5	F5	A2	1·5	1·3 *e*
32 Eridani	4·9	6·3	0·005	−1·4	0·0	G5	B?	1·4	3·5 *e*
θ^2 Orionis	5·2 A	6·5 B	0·005	−1·1	0·8	B1	B1	1·3	0
1 Arietis	6·2	7·4	0·010 S	1·2	2·4	Go	A	1·2	2·0 *e*
14 Lyncis	5·9	7·1	0·009 H	0·7	1·9	F5	A2	1·2	1·3 *e*
γ Leonis	2·6	3·8	0·009	−2·6	−1·4	G8	G5	1·2	0·3 *e*
β.G.C. 8398	6·0	7·1	0·009 H	0·8	1·9	F2	Ao	1·1	1·2 *e*
Lal. F. 52	6·8	7·8	0·003	−0·8	0·2	Go	Go	1·0	0·0
γ Delphini	4·5	5·5	0·026	1·6	2·6	K1	F6	1·0	1·5 *e*
δ Cephei	4·3 (var.)	5·3	0·006	−1·8	0·8	Go	A	1·0	2·0 *e*
δ Serpentis	4·2	5·2	0·013	0·2	1·2	A7	A8	1·0	0·1 *l*
19 Lyncis	5·6	6·5	0·005	−0·9	0·0	B8	A	0·9	0·2 *l*
π Aquilæ	6·0	6·8	0·008 H	0·5	1·3	F2	A2	0·8	1·0 *e*
O.A. 13803	8·0	8·5	0·002	−0·5	0·0	Go	Go	0·5	0
β.G.C. 4839	6·1	6·6	0·007 S	0·4	0·9	G9	Ko	0·5	0·1 *l*
ϵ^2 ϵ^1 Lyræ	4·5	4·7	0·005	−2·0	−1·8	A5	A3	0·2	0·2 *e*
95 Herculis*	5·1	5·2	0·008 S	−0·4	−0·3	G5	A3	0·1	2·2 *e*

* Spectra as given by Campbell, *Stellar Motions*, p. 271. *H.C. Annals*, 56, 248, states: "This [composite] spectrum appears to be of Class A3, with a fainter spectrum of class G5 superposed."

In column ΔT, *l* = "later," *e* = "earlier."

In column π, S = spectroscopic, H = hypothetical.

No attempt is made to allow in ΔM for either component being a spectroscopic binary.

TABLE II.

Systems composed of Dwarf Stars.

Star.	ma.	mb.	π.	Ma.	Mb.	Ta.	Tb.	ΔM.	ΔT.
α Gem. and Comes	2·0	9·0	0·079	1·5	8·5	Ao	Md	7·0	4·0 *l*
μ Herculis	3·5	9·7	0·095	3·4	9·6	G5	Mb	6·2	1·5 *l*
η Cassiop.	3·7	7·4	0·186	5·1	8·8	F8	K5	3·7	1·7 *l*
17 Cygni	5·0	8·1	0·045	3·3	6·4	F6	K4	3·1	1·8 *l*
β.G.C. 7060	5·8	8·9	0·194	7·2	10·3	K3	Ma	3·1	0·7 *l*
Pi 10^h, 135–137	5·3	8·2	0·014	1·0	3·9	F	G5	2·9	1·5 *l*
Boss 441	4·8	7·7	0·022 S	1·5	4·4	A5	Go	2·9	1·5 *l*
γ Leporis	3·8	6·4	0·149	4·7	7·3	F6	K5	2·6	1·9 *l*
γ Ceti	3·6	6·2	0·049	2·1	4·7	A	F3	2·6	1·3 *l*
ξ^1 ξ^2 Scorpii	4·8 A	7·2 C	0·046	3·1	5·5	F5	G7	2·4	1·2 *l*
β.G.C. 5858	6·3	8·6	0·038	4·2	6·5	F8	K4	2·3	1·6 *l*
β.G.C. 6842	5·1	7·4	0·018	1·4	3·7	Ao	Fo	2·3	1·0 *l*
μ^1 μ^2 Boötis	4·5	6·7	0·046	2·8	5·5	Fo	Ko	2·2	2·0 *l*
ν Scorpii	4·3	6·5	0·025	1·3	3·5	B3	A	2·2	0·7 *l*
ξ Boötis	4·8	6·8	0·178	6·0	8·0	G6	K4	2·0	0·8 *l*
κ Boötis	4·6	6·6	0·020	1·1	3·1	A5	A5 (?)	2·0	0·0
Boss 5998	5·3	7·2	0·039 S	3·3	5·2	G5	K2	1·9	0·7 *l*
Boss 285	5·2	7·0	0·04 S	3·2	5·0	F2	G9	1·8	1·7 *l*
ξ Cephei	4·7	6·5	0·029	2·0	3·8	A8	F8	1·8	1·0 *l*
D.M. 1657, 1658	5·3	7·1	0·02	1·8	3·6	Fo	Go	1·8	1·0 *l*
70 Ophiuchi	4·3	6·0	0·187	5·7	7·4	Ko	K6	1·7	0·6 *l*
Σ 711	7·6	9·2	0·030 S	5·0	6·6	G1	K5	1·6	1·4 *l*
ζ Urs. Maj.	2·4	4·0	0·040	0·4	2·0	AP	A2	1·6	0·2 *l*
α Centauri	0·3	1·7	0·759	4·7	6·1	G6	K4	1·4	0·8 *l*
36^1 Ophiuchi 30 Scorpii	5·3	6·7	0·140	6·5	7·9	K1	K4	1·4	0·3 *l*
ϵ Equulei	6·0 A	7·4 C	0·019	2·4	3·8	F1	F2	1·4	0·1 *l*
β.G.C. 7268	6·8	8·1	0·046 S	5·1	6·4	G8	K2	1·3	0·4 *l*
υ^1 Cancri	7·1	8·3	0·032 H	4·6	5·8	A3	G	1·2	1·7 *l*
χ Aquilæ	5·6	6·8	0·023 S	2·4	3·6	F5	A	1·2	1·5 *e*
β.G.C. 4677	6·0	7·2	0·024	2·9	4·1	F8	G4	1·2	0·6 *l*
ψ Draconis	4·9	6·1	0·030	2·3	3·5	F3	F7	1·2	0·4 *l*
β.G.C. 12234	8·3	9·4	0·019	4·7	5·8	F5	G2	1·1	0·7 *l*
83 Leonis	6·5	7·6	0·023	3·3	4·4	G9	K4	1·1	0·5 *l*
β.G.C. 12755	6·4	7·5	0·036	4·2	5·3	G4	G7	1·1	0·3 *l*
ζ Piscium	5·6	6·5	0·019	2·0	2·9	A5	F8	0·9	1·3 *l*
β.G.C. 11051	7·1	8·0	0·017 S	3·3	4·2	F6	F7	0·9	0·1 *l*
D.M. 561, 560	6·6	7·5	0·013	2·2	3·1	B9	Ao	0·9	0·1 *l*
σ Cor. Bor.	5·8	6·7	0·044	4·0	4·9	F8	F9	0·9	0·1 *l*

TABLE II.—*continued.*

Star.	ma.	mb.	π.	Ma.	Mb.	Ta.	Tb.	ΔM.	ΔT.
α Piscium	4·3	5·2	0·042 H	2·4	3·3	A2	A	0·9	0·2 *e*?
30 Arietis	6·6	7·4	0·029	3·9	4·7	F3	F5	0·8	0·2 *l*
44 Boötis	5·3	6·1	0·069 S	4·5	5·3	Go	G2	0·8	0·2 *l*
β.G.C. 322	7·9	8·7	0·030	5·3	6·1	G3	G3	0·8	0
Boss 5881	6·8 A	7·5 B	0·032 S	4·3	5·0	G3	G5	0·7	0·2 *l*
61 Cygni	5·6	6·3	0·307	8·0	8·7	K7	K8	0·7	0·1 *l*
Boss 3983, 3986	6·9	7·6	0·021	3·5	4·2	G3	G5	0·7	0·2 *l*
β.G.C. 10504	7·5	8·2	0·016	3·5	4·2	G8	G6	0·7	0·2 *e*
β.G.C. 7362	6·9	7·6	0·022	3·6	4·2	G5	G5	0·6	0·0
β.G.C. 1854	8·6	9·2	0·030	6·0	6·6	K0	K2	0·6	0·2 *l*
β.G.C. 9114	8·2	8·7	0·023	5·0	5·5	G4	K0	0·5	0·6 *l*
ξ Urs. Majoris	4·4	4·9	0·158	5·4	5·9	F9	G2	0·5	0·3 *l*
β.G.C. 11741	8·3	8·8	0·032 S	5·8	6·3	G9	K2	0·5	0·3 *l*
W.B. 15^{h}, 716, 720	7·4	7·9	0·046	5·7	6·2	K3	K4	0·5	0·1 *l*
β.G.C. 4815	8·7	9·1	0·086	8·4	8·8	K6	K9	0·4	0·3 *l*
O.A.S. 14318, 14320	9·2	9·6	0·020	5·7	6·1	G4	G5	0·4	0·1 *l*
β.G.C. 4929	7·3	7·7	0·019	3·7	4·1	F3	F4	0·4	0·1 *l*
ζ Cancri	5·6 A	6·0 C	0·038	3·5	3·9	F9	Go	0·4	0·1 *l*
Σ 618	7·0	7·3	0·028 S	4·2	4·5	G2	G5	0·3	0·3 *l*
$\zeta^2\,\zeta^1$ Reticuli	5·2	5·5	0·10	5·2	5·5	F8	Go	0·3	0·2 *l*
β.G.C. 10281	7·0	7·3	0·004 H	−0·3	0·0	A0	A0	0·3	0·0
β.G.C. 6641	7·9	8·2	0·069 S	7·1	7·4	K6	K6	0·3	0·0
β.G.C. 6090	7·5	7·8	0·034 S	5·1	5·4	G8	G8	0·3	0·0
β.G.C. 9137	6·6	6·8	0·045	4·9	5·1	G6	G4	0·2	0·2 *e*
ζ Aquarii	4·4	4·6	0·036	2·2	2·4	F2	F2	0·2	0·0
β.G.C. 5695	8·8	8·9	0·048	7·2	7·3	K7	Ma	0·1	0·3 *l*
16 Cygni	6·3	6·4	0·038	4·2	4·3	G2	G3	0·1	0·1 *l*
β.G.C. 4972	7·9	8·0	0·152	8·8	8·9	K2	K2	0·1	0·0
ν Draconis	4·9	5·0	0·014	0·7	0·8	A5	A5	0·1	0·0
β.G.C. 5960	8·4	8·4	0·028 S	5·6	5·6	G4	G6	0	0·2 *l*
γ Virginis	3·7	3·7	0·068	2·9	2·9	Fo	F1	0	0·1 *l*
$36^1\,36^2$ Ophiuchi	5·3	5·3	0·140	6·5	6·5	K1	K2	0	0·1 *l*
β.G.C. 12675	6·0	6·0	0·035 S	3·7	3·7	F9	Go	0	0·1 *l*
μ Draconis	5·8	5·8	0·027 S	2·9	2·9	F5	F5	0	0
β.G.C. 8986	8·5	8·5	0·030 H	5·9	5·9	G5	G5	0	0
49 Serpentis	7·6	7·6	0·078	7·1	7·1	G9	G9	0	0
65 Piscium	6·3	6·3	0·010	1·3	1·3	Fo	Fo	0	0

In ΔT, *l* = "later," *e* = "earlier."
In π, S = spectroscopic, H = hypothetical.
No attempt has been made to allow in ΔM for either component being a spectroscopic binary.

TABLE III.

Approximations to Parallax.

Star.	1900. R.A.	1900. Dec.	Ma.	Mb.	Separation.	Ta.	Tb.	Estimated. Ma.	Mb.	πa.	πb.	π.
	h m	° ′			″					″	″	″
— Tauri	3 27·5	+23 1	8·2	8·5	70·8	A2	G	1·0*d*	4·5*d*	0·004	0·016	0·010
γ Volantis	7 9·6	−70 20	3·9	5·8	13·3	Ko	G	0·0*g*	4·5*d*	0·017	0·055	0·036
α^2 Libræ	14 45·3	−15 38	2·9	5·1	231·0	A2	F5	1·0*d*	3·5*d*	0·042	0·048	0·045
b Draconis	18 22·5	+58 45	4·9	7·6	88·8	A2	F	1·0*d*	2·5*d*	0·017	0·010	0·013
— Lyræ	18 51·2	+33 51	6·1	7·8	45·5	G	A	−0·5*g*	0·7*d*	0·005	0·004	0·005
57 Aquilæ	19 49·3	− 8 29	5·8	6·5	35·7	B3	A	−1·5*g*	−1·3*g*	0·003	0·003	0·003
— Cephei	22 18·8	+66 12	7·1	8·1	4·1	G	A	−0·5*g*	0·7*d*	0·003	0·003	0·003
— Cassiop.	23 57·5	+65 33	6·0	7·5	15·0	F5	A2	−0·7*g*	1·0*d*	0·005	0·005	0·005
Stars in Præsepe (*Messier* 44 *Cancri*).												
	h m	° ′			″					″	″	″
— Cancri	8 34·2	+19 54	6·9	7·7	92·9	A	G5	0·7*d*	5·4*d*	0·006	0·035	0·020
— ,,	,,	,,	6·9	7·2	45·0	A	A	0·7*d*	0·7*d*	0·006	0·006	0·006
— Cancri	8 34·6	+20 1	6·4	6·9	63·4	G	A2	−0·5*g*	1·0*d*	0·004	0·007	0·006
— ,,	,,	,,	6·4	9·1	82·5	G	G	−0·5*g*	4·5*d*	0·004	0·012	0·008
ε Cancri	8 34·7	+19 54	6·3	7·4	134·2	A2	A2	1·0*d*	1·0*d*	0·009	0·005	0·007
											Mean	0·010

In columns 9 and 10, *g* = Giant, *d* = Dwarf.

Note on Comet 1906 *VI.* (*Metcalf*). By G. Merton, M.C., M.A.

This faint periodic comet was discovered by Metcalf on 1906 November 14, and was then observed for about eight weeks only. No reliable orbit based on all available observations has yet been published, however.

It was unfavourably placed at its return in 1914, but another return was due this year. It seemed desirable, therefore, to investigate the orbit and to obtain some information as to the effect of the close approach of the comet to Jupiter in 1911. Soon after commencing the work, some unpublished elements by Professor Bianchi became available through the kindness of Dr. A. C. D. Crommelin, who communicated them. These were found to satisfy two provisional normal places very satisfactorily, and were therefore used in the computation of the perturbations without modification.

The elements given by Professor Bianchi were as follows:—

Epoch 1906 *November* 17·5, *Berlin M.T.*

M	4° 54′ 4″·7		
ω	199° 59′ 47″·6	1910·0	
Ω	194° 33′ 15″·1		
i	14° 37′ 27″·7		
φ	35° 43′ 57″·2		
		μ	456″·4983
		log *a*	·593714
		log *q*	·212805
		U	7·7781 years
		T	1906 October 9·8477, Berlin M.T.

The deduced period must, however, be subject to considerable uncertainty owing to the shortness of the observed arc. For this reason the perturbations by Jupiter only were computed for the interval from 1908 July 21 to 1914 January 11, during which the comet was within 3·2 units of Jupiter, by the Hansen-Tietjen polar co-ordinate method as given by Oppolzer and Bauschinger, but using eighty-day intervals and five-figure accuracy only. A general check on the work was furnished by an independent calculation by an approximate method given by Oppolzer,* which, however, only gives a very rough indication in such cases as this, owing to the neglect of all but the first-order terms.

The nearest approach to Jupiter, 0·86 units, took place on 1911 September 15. New osculating elements were computed for 1914 January 11, and this work was additionally checked by computing the equatorial rectangular co-ordinates for 1913 October 13 and 1914 January 11, both with these new elements and with the Gaussian equations of the original elements in the modified form by which the perturbations in the polar co-ordinates were included.

The dates of perihelion passage were found to be 1914 June 11·3, 1922 March 3, and 1929 November 23. Unfortunately the first two were unfavourable, but that of 1929 will be very favourable provided that the date given is not too greatly in error.

The perturbations over the interval considered are given below, expressed in terms of the usual elements, together with the new elements resulting from them, reduced to 1929·0, in the hope that a search ephemeris based on them may then lead to the recovery of this comet.

$M - M_0$	+4° 39′	T	1929 Nov 23
$\omega - \omega_0$	+3° 24′	ω	203° 24′
$\Omega - \Omega_0$	−4° 59′	Ω	189° 50′
$i - i_0$	−1° 9′	*i*	13° 28′
$\phi - \phi_0$	+0° 40′	φ	36° 24′
$\mu - \mu_0$	+2″·5	μ	459″·0
		log *a*	·5921
		log *q*	·2013

Woldingham:
1922 *April* 10.

* *Sitzungsberichte Kais. Acad.*, Vienna, **62**, 655.

Approach of Brooks's Periodic Comet to Jupiter. By Professor C. Pokrovsky, Pulkovo Observatory.

(*Communicated in abstract by A. C. D. Crommelin.*)

I deduce from Chandler's elements that this comet (1889 V.) will approach Jupiter on June 12 next within a distance of 0·285 units. It may not be quite useless to make photographic search, seeing that comets 1889 I. and 1889 II. were observed by Barnard at distances from the earth of 6·00 and 4·07 respectively. Wolf's comet will also be near Jupiter at the end of 1922 and beginning of 1923, and may suffer serious alteration of its orbit.

Determination of the Elements of the Orbit of a Planet or a Comet by means of the Variation of Two Geocentric Distances. By M. Subbotin.

(*Communicated by A. C. D. Crommelin.*)

In the method of the variation of two geocentric distances in the usual form it is necessary to compute in every assumption, made upon the geocentric distances for the extreme places, not only all the elements of the orbit but also the geocentric co-ordinates for the date of any intermediate observation. A new form of Lambert's theorem permits us to present this method in the following more practicable form :—*

1. Let x, y, z, x', y', z', r, r' be the heliocentric co-ordinates and the radii-vectores of a heavenly body at the times t and t' respectively.

In analogy with the well-known formula

$$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2=k^2(1+m)\left(\frac{2}{r}-\frac{1}{a}\right)$$

we may write

$$\frac{(x-x')^2+(y-y')^2+(z-z')^2}{(t-t')^2}=4k^2(1+m)\left(\frac{\tau}{r+r'}-\frac{1}{4a}\right),$$

or, if we put

$$(x-x')^2+(y-y')^2+(z-z')^2=s^2, \quad 2k\sqrt{1+m}(t-t')=\theta,$$

we get

$$(1) \quad . \quad . \quad . \quad . \quad . \quad . \quad \frac{1}{4a}=\frac{\tau}{r+r'}-\frac{s^2}{\theta^2}.$$

---

* This form was first given by the writer in 1919 (*Annales de l'Institut Polytechnique du Don*, t. viii.).

From this equation we have

$$\tau = \frac{r+r'}{4a} - \frac{s^2}{\theta^2}(r+r'),$$

or

$$\tau = \mathrm{R} + \sigma^2 \mathrm{R}^3 \frac{(4a)^3}{\theta^2},$$

putting

$$\frac{r+r'}{4a} = \mathrm{R}, \quad \frac{s}{r+r'} = \sigma.$$

It is of great importance that τ can be expressed by means of only two quantities, R and σ. So, for the case of elliptic motion, we have

$$\tau = \mathrm{R} + \frac{16\mathrm{R}^3\sigma^2}{(\epsilon - \sin\epsilon + \delta - \sin\delta)^2},$$

$$\sin\frac{\epsilon}{2} = \sqrt{\mathrm{R}(1+\sigma)}, \quad \sin\frac{\delta}{2} = \sqrt{\mathrm{R}(1-\sigma)},$$

where $\sin\epsilon$ and $\sin\delta$ have the same sign when the two foci are on the same side of the chord joining the two positions of the body, and opposite signs when the chord passes between the foci. For parabolic motion $\mathrm{R} = 0$ and in the formula (1) we have

$$\tau = \left(\frac{3\sigma}{(1+\sigma)^{\frac{3}{2}} - (1-\sigma)^{\frac{3}{2}}}\right)^2.$$

In the case of hyperbolic motion ($a < 0$) the corresponding formulæ are

$$\tau = \mathrm{R} - \frac{16\mathrm{R}^3\sigma^2}{(\sinh\epsilon - \epsilon + \delta - \sinh\delta)^2},$$

$$\sinh\frac{\epsilon}{2} = \sqrt{-\mathrm{R}(1+\sigma)}, \quad \sinh\frac{\delta}{2} = \sqrt{-\mathrm{R}(1+\sigma)}.$$

In all cases the values of τ may be tabulated with the arguments R and σ, or $lg\mathrm{R}$ and σ. By means of such tables it is very easy to find by trial the value of a when $r+r'$, s, and $t-t'$ are known, because τ changes slowly, being little different from unity. I hope to publish the necessary tables in the future.

The form (1) of Lambert's theorem has also some theoretical interest, being available for generalisations.

2. Let t, t_1, t_2 ($t_1 < t < t_2$) be the times of observation, α, α_1, α_2, δ, δ_1, δ_2 the corresponding right ascensions and declinations (or longitudes and latitudes), and ρ, ρ_1, ρ_2 the distances of the body from the earth, and let

$$\lambda = \cos\delta\cos\alpha, \quad \mu = \cos\delta\sin\alpha, \quad \nu = \sin\delta, \ldots$$

Then we shall have

$$(2)\quad \begin{cases} x=\rho\lambda-X, & x_1=\rho_1\lambda_1-X, & x_2=\rho_2\lambda_2-X_2, \\ y=\rho\mu-Y, & y_1=\rho_1\mu_1-Y, & y_2=\rho_2\mu_2-Y_2, \\ z=\rho\nu-Z, & z_1=\rho_1\nu_1-Z, & z_2=\rho_2\nu_2-Z_2, \end{cases}$$

where X, Y, Z, . . . are the corresponding sun's co-ordinates.

Substituting these values in the relation

$$x(y_1z_2-y_2z_1)-y(x_1z_2-x_2z_1)+z(y_2x_1-y_1x_2)=0,$$

and putting

$$\lambda_{12}=\mu_1\nu_2-\mu_2\nu_1,\ l_{12}=\nu_1Y_2-\mu_1Z_2,\ l_{21}=\mu_2Z_1-\nu_2Y_1,\ X_{12}=Y_1Z_2-Y_2Z_1,$$
$$\mu_{12}=\lambda_1\nu_2-\lambda_2\nu_1,\ m_{12}=\nu_1X_2-\lambda_1Z_2,\ m_{21}=\lambda_2Z_1-\nu_2X_1,\ Y_{12}=X_1Z_2-X_2Z_1,$$
$$\nu_{12}=\lambda_1\mu_2-\lambda_2\mu_1,\ n_{12}=\mu_1X_2-\lambda_1Y_2,\ n_{21}=\lambda_2Y_1-\mu_2X_1,\ Z_{12}=X_1Y_2-X_2Y_1$$

$$\begin{aligned} A&=\lambda\lambda_{12}-\mu\mu_{12}+\nu\nu_{12}, & M&=X\lambda_{12}-Y\mu_{12}+Z\nu_{12}, \\ B&=\lambda l_{12}-\mu m_{12}+\nu n_{12}, & N&=Xl_{12}-Ym_{12}+Zn_{12}, \\ C&=\lambda l_{21}-\mu m_{21}+\nu n_{21}, & P&=Xl_{21}-Ym_{21}+Zn_{21}, \\ D&=\lambda X_{12}-\mu Y_{12}+\nu Z_{12}, & Q&=XX_{12}-YY_{12}+ZZ_{12}, \end{aligned}$$

the result is

$$(3)\quad \cdot\quad \cdot\quad \cdot\quad \cdot\quad =\frac{M\rho_1\rho_2+N\rho_1+P\rho_2+Q}{A\rho_1\rho_2+B\rho_1+C\rho_2+D}.$$

3. The problem of determining the elements of an orbit from three observed places is equivalent to the resolution of a system of nine equations with nine unknown quantities: six elements and three geocentric distances ρ, ρ_1, ρ_2. By using Lambert's theorem we obtain from this system three equations of the form

$$(4)\quad \cdot\quad \phi(\rho_1, \rho_2, a)=0,\quad \phi_1(\rho_1, \rho_2, a)=0,\quad \phi_2(\rho_1, \rho_2, a)=0,$$

because the radii-vectores r, r_1, r_2 and the chord s, s_1, s_2 may be expressed in terms of ρ_1 and ρ_2 by means of the formulæ (2), (3), and the relations

$$r=\sqrt{x^2+y^2+z^2},\quad r_1=\sqrt{x_1^2+y_1^2+z_1^2},\quad r_2=\sqrt{x_2^2+y_2^2+z_2^2},$$
$$s^2=(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2,\quad s_1^2=(x_1-x)^2+(y_1-y)^2+(z_1-z)^2,$$
$$s_2^2=(x_2-x)^2+(y_2-y)^2+(z_2-z)^2.$$

But the numerical resolution of the system (4) with three unknown quantities being not practicable, we shall reduce the number of the unknown quantities. It is easy to do if we use Lambert's theorem in the form given above. In this case we may regard the equations (4) as presented in the form

$$\frac{1}{4a}=\psi(\rho_1, \rho_2),\quad \frac{1}{4a}=\psi_1(\rho_1, \rho_2),\quad \frac{1}{4a}=\psi_2(\rho_1, \rho_2),$$

and equating these expressions for $\frac{1}{4a}$, we get

$$\psi(\rho_1, \rho_2)-\psi_1(\rho_1, \rho_2)=0,\quad \psi(\rho_1, \rho_2)-\psi_2(\rho_1, \rho_2)=0.$$

The resolution of this system presents no difficulty when sufficiently approximate values of ρ_1 and ρ_2 are known.

In many cases it is convenient to use the following system :—

$$A(\rho_1, \rho_2) = 0, \quad B(\rho_1, \rho_2) = 0,$$

where $$A(\rho_1, \rho_2) = \log \frac{\psi(\rho_1, \rho_2)}{\psi_1(\rho_1, \rho_2)}, \quad B(\rho_1, \rho_2) = \log \frac{\psi(\rho_1, \rho_2)}{\psi_2(\rho_1, \rho_2)}.$$

4. To illustrate the application of this method let us take the following observations of Ekard, 694, which were used by P. Harzer for trial of his method for determining the orbit of a planet from three observations (*Astr. Nach.*, No. 4674) :—

1909 Nov. 15. $t' = -0.616\,9446$, $\lambda_1 = +0.544\,137$, $\mu_1 = +0.838\,315$,
Dec. 21. $t' = -0.000\,1486$, $\lambda = +0.641\,195$, $\mu = +0.760\,004$,
1910 Jan. 28. $t' = +0.652\,1595$, $\lambda_2 = +0.606\,641$, $\mu_2 = +0.783\,222$,

$\nu_1 = -0.033\,817$, $X_1 = -0.595\,495$, $Y_1 = -0.790\,145$, $Z_1 = 0$,
$\nu = -0.106\,126$, $X = -0.011\,102$, $Y = -0.983\,757$, $Z = 0$,
$\nu_2 = -0.136\,196$, $X_2 = +0.606\,051$, $Y_2 = -0.776\,506$, $Z_2 = 0$.

These places are referred to the ecliptic and mean equinox of 1910·0, and are freed from aberration. The intervals of time computed from Dec. 21, $8^h\,58^m\,12^s.4$, Berlin M.T., are multiplied by k. The sun's coordinates are reduced to the corresponding *loci ficti*.

In the present case we obtain

$$\rho = \frac{51750\rho_1\rho_2 + 20453\rho_1 + 78592\rho_2}{67512\rho_1\rho_2 + 66346\rho_1 + 59902\rho_2 + 99894};$$

and from

$$\begin{aligned} \theta &= 2k(t_2 - t_1), & \theta_1 &= 2k(t - t_1), & \theta_2 &= 2k(t_2 - t), \\ &= 2(t_2' - t_1'), & &= 2(t' - t_1'), & &= 2(t_2' - t'), \end{aligned}$$

we find

$$\theta^{-2} \ldots 9.19\,0946, \quad \theta_1^{-2} \ldots 9.76\,9034, \quad \theta_2^{-2} \ldots 9.81\,7658.$$

If we suppose that the approximate values are $\rho_1 = 1.22 \ldots$, $\rho_2 = 2.04 \ldots$, we may begin with the following three hypotheses :—

	$\rho_1 =$	$\rho_2 =$	$\rho =$	r_1	r	r_2
I.	1·23	2·05	1·50 2637,	0·34 591,	0·36 995,	0·39 475
II.	1·22	2·05	1·50 2349,	0·34 395,	0·36 990,	0·39 475
III.	1·23	2·04	1·49 6904,	0·34 591,	0·36 892,	0·39 313

	s^2	s_1^2	s_2^2
I.	9·88 332,	9·28 064,	9·28 698.
II.	9·88 493,	9·28 536,	9·28 697.
III.	9·88 235,	9·27 920,	9·28 623.

For the first hypothesis, from

$$\frac{1}{4a} = \frac{\tau}{r_1 + r_2} - \frac{s^2}{\theta^2},$$

where we put $\tau = 1$, we obtain

$$\frac{1}{4a} \ldots 8{\cdot}97\ 376.$$

With the resulting value $\log R = \log \frac{r_1 + r_2}{4a} = 9{\cdot}64\ 581$ and $\sigma = \frac{s}{r_1 + r_2} = 0{\cdot}18604$ as arguments, we find from the tables above mentioned, $\log \tau = 9{\cdot}99\ 826$, and then we get

$$\frac{1}{4a} \ldots 8{\cdot}96\ 982, \quad R \ldots 9{\cdot}64\ 187.$$

By means of this value of $\log R$ we derive $\log \tau = 9{\cdot}99\ 831$, and hence finally,

$$\frac{1}{4a} \ldots 8{\cdot}96\ 994.$$

We might, of course, have begun with a more exact value of τ than $\tau = 1$. For instance, we may retain for the following hypothesis the value just found as a first approximation.

The application of the same process gives

$$\log\left(\frac{\tau_1}{r + r_1} - \frac{s_1^2}{\theta_1^2}\right) = 8{\cdot}97\ 130, \log\left(\frac{\tau_2}{r + r_2} - \frac{s_2^2}{\theta_2^2}\right) = 8{\cdot}96\ 964,$$

so that

$$A(\rho_1, \rho_2) = -0{\cdot}00\ 136, \ B(\rho_1, \rho_2) = +0{\cdot}00\ 030.$$

In the following table are given the results and some intermediate numbers for all three hypotheses:—

	$\sigma =$	$\sigma_1 =$	$\sigma_2 =$	τ	τ_1	τ_2
I.	0·18 604	0·09 576	0·09 119	9·99 831	9·99 959	9·99 956
II.	0·18 679	0·09 650	0·09 119	9·99 830	9·99 960	9·99 956
III.	0·18 620	0·09 572	0·09 139	9·99 828	9·99 958	9·99 955

I.	$A(\rho_1, \rho_2) = -0{\cdot}00\ 136$,	$B(\rho_1, \rho_2) = +0{\cdot}00\ 030$
II.	$= +0{\cdot}00\ 271$,	$= +0{\cdot}00\ 026$
III.	$= -0{\cdot}00\ 142$,	$= -0{\cdot}00\ 075$

Let us suppose that the squares and products of the differences between the true and the assumed values of ρ_1 and ρ_2 may be neglected. Then, putting

$$\rho_1 = 1{\cdot}23 - 0{\cdot}01\lambda'', \quad \rho_2 = 2{\cdot}05 - 0{\cdot}01\lambda''',$$

and determining λ'', λ''' from the equations

$$\begin{aligned} 30\lambda' + 26\lambda'' - 45\lambda''' &= 0, \\ 136\lambda' - 271\lambda'' + 142\lambda''' &= 0, \\ \lambda' + \lambda'' + \lambda''' &= 1, \end{aligned}$$

we obtain

$$\text{IV.} \quad \rho_1 = 1{\cdot}22\ 660, \quad \rho_2 = 2{\cdot}04\ 618.$$

These values lead to

$$\text{IV. } A = -0{\cdot}00\,0074, \quad B = +0{\cdot}00\,0046.$$

Using now the numbers found in the hypothesis IV., I. and III., we obtain, in the same manner, the following values:—

$$\rho_1 = 1{\cdot}22\,640, \quad \rho_2 = 2{\cdot}04\,554,$$

which differ very little from those found by P. Harzer in the paper above quoted:

$$\rho_1 = 1{\cdot}22\,637, \quad \rho_2 = 2{\cdot}04\,554.$$

Corresponding elements represent the second place of the planet with all desired precision.

5. It is interesting to note that the numbers ρ_1, ρ_2, which satisfy the equations

$$A(\rho_1, \rho_2) = 0, \quad B(\rho_1, \rho_2) = 0,$$

do not always lead to a system of elements representing the observations used. The resolution of these equations gives us the co-ordinates of three points P_1, P, P_2, lying in a plane with the sun's centre, and such that a particle, moving round the sun under Kepler's laws, describes the arcs P_1P_2, PP_2, P_1P in the given times $t_2 - t_1$, $t_2 - t_1$, $t - t_1$. But it may happen that the last condition is satisfied when the particle describes the arcs belonging to distinct ellipses, and then the elements of the orbit, found from the co-ordinates of P_1 and P_2, or P_1 and P, or P and P_2, will be different.

In practice such values of ρ_1 and ρ_2 may be encountered only in applying the explained method to the determination of an entirely unknown orbit.

To illustrate this case, let us take the following observations of Nausicaa, 192:—*

	Gr. M.T.	α 1912·0.	
1912 March	20·30 0387	107° 58′ 4·1″	+27° 18′ 7·8″,
	25·29 4727	108° 49′ 5·2″	+26° 55′ 22·3″,
	47·34 5028	113° 57′ 53·7″	+25° 9′ 56·3″,

the corresponding topocentric co-ordinates of the sun being

$X_1 = +0{\cdot}99\,6410$, $Y_1 = -0{\cdot}00\,2933$, $Z_1 = -0{\cdot}00\,1224$.
$X = +0{\cdot}99\,4342$, $Y = +0{\cdot}07\,6234$, $Z = +0{\cdot}03\,3055$.
$X_2 = +0{\cdot}89\,8926$, $Y_2 = +0{\cdot}41\,0522$, $Z_2 = +0{\cdot}17\,8056$.

In this case we have

$\lambda_1 = -0{\cdot}27\,4118$, $\mu_1 = +0{\cdot}84\,5264$, $\nu_1 = +0{\cdot}45\,8683$
$\lambda = -0{\cdot}28\,7604$, $\mu = +0{\cdot}84\,3958$, $\nu = +0{\cdot}45\,2790$
$\lambda_2 = -0{\cdot}36\,7623$, $\mu_2 = +0{\cdot}82\,7060$, $\nu_2 = +0{\cdot}42\,5236$

θ^{-2} . . . 0·06 2615, θ_1^{-2} . . . 1·52 9821, θ_2^{-2} . . . 0·23 9948,

$$\rho = \frac{210\,000\rho_1\rho_2 - 312\,635\rho_1 - 52\,955\rho_2 + 204}{1\,585\rho_1\rho_2 + 50\,483\rho_1 + 155\,063\rho_2 - 357\,514}.$$

* *Bulletin Astronomique*, t. 29, p. 441.

I have begun the calculation with $\rho_1 = 1{\cdot}9$, $\rho_2 = 2{\cdot}6$, and after six hypotheses I have obtained the values

$$\rho_1 = 2{\cdot}36\ 3478, \quad \rho_2 = 2{\cdot}79\ 3111,$$

satisfying very closely the equations

$$A(\rho_1, \rho_2) = 0, \quad B(\rho_1, \rho_2) = 0.$$

With these values (and $\rho = 2{\cdot}43\ 3109$) we may correct the times of observation for the time of aberration.

Thus we get

$$t_1 = 20{\cdot}28\ 6749, \quad t = 25{\cdot}28\ 0688, \quad t_2 = 47{\cdot}32\ 8911,$$

and hence

$$\theta^{-2} \ldots 0{\cdot}06\ 2694, \quad \theta_1^{-2} \ldots 1{\cdot}52\ 9890, \quad \theta_2^{-2} \ldots 0{\cdot}24\ 0030.$$

Then, recomputing ρ_1 and ρ_2, we have

ρ_1	=	$2{\cdot}36\ 3464$,	ρ =	$2{\cdot}43\ 3094$,	ρ_2 = $2{\cdot}79\ 3092$
$r_1 \ldots$		$0{\cdot}44\ 8381$,	$r \ldots$	$0{\cdot}44\ 9396$,	$r_2 \ldots$ $0{\cdot}46\ 0481$
$s^2 \ldots$		$8{\cdot}97\ 8492$,	$s_1^2 \ldots$	$7{\cdot}52\ 0308$,	$s_2^2 \ldots$ $8{\cdot}80\ 0402$

$$A(\rho_1, \rho_2) = 0{\cdot}00\ 0000, \quad B(\rho_1, \rho_2) = -0{\cdot}00\ 0001,$$
$$a \ldots 0{\cdot}58\ 0988.$$

From these values, by means of the equations

$$\sin\frac{\epsilon}{2} = \sqrt{R(1+\sigma)} = \sqrt{\frac{r_1 + r_2 + s}{4a}}, \quad \sin\frac{\delta}{2} = \sqrt{R(1-\sigma)} = \sqrt{\frac{r_1 + r_2 - s}{4a}},$$

$$(5) \quad \ldots \quad \tfrac{1}{2}(u_2 - u_1) = \tfrac{1}{2}(\epsilon - \delta),$$

$$\begin{cases} 2ae \sin \tfrac{1}{2}(u_2 + u_1) = (r_2 - r_1) \operatorname{cosec} \tfrac{1}{2}(u_2 - u_1) \\ 2ae \cos \tfrac{1}{2}(u_2 + u_1) = (2a - r_2 - r_1) \sec \tfrac{1}{2}(u_2 - u_1), \end{cases}$$

we compute the eccentricity and the eccentric anomalies for the extreme places:

$$\tfrac{1}{2}(u_2 - u_1) = 2^\circ\ 23'\ 52{\cdot}6'', \ u_2 = 46^\circ\ 55'\ 30{\cdot}0'', \ u_1 = 42^\circ\ 7'\ 44{\cdot}8'', \ e = 0{\cdot}35\ 4793.$$

Next, from

$$M_1 = u_1 - e \sin u_1, \quad M_2 = u_2 - e \sin u_2,$$

$$n = k'' a^{-\frac{3}{2}} = \frac{M_2 - M_1}{t_2 - t_1},$$

we obtain

$$M_1 = 28^\circ\ 29'\ 34''{\cdot}6, \ M_2 = 32^\circ\ 4'\ 33''{\cdot}9, \ n = 477''{\cdot}008.$$

Before proceeding to the determination of the other elements it is advisable to compute r from

$$M_1 + n(t - t_1) = u - e \sin u, \ r = a(1 - e \cos u).$$

In the present case we obtain

$$u = 43^\circ\ 1'\ 29''\cdot4,\ r\ \ldots\ 0\cdot45\,0587,$$

which value differs greatly from $r\ \ldots\ 0\cdot44\,9396$. The equations (5) applied to the first and second places, and to the second and third, give, respectively,

$\frac{1}{2}(u-u_1) = 0^\circ\ 26'\ 54''\cdot4,\ u = 23^\circ\ 13'\ 11''\cdot0,\ u_1 = 22^\circ\ 19'\ 22''\cdot2,\ e = 0\cdot28\,4444.$
$\frac{1}{2}(u_2-u) = 1^\circ\ 57'\ 9''\cdot9,\ u_2 = 49^\circ\ 59'\ 5''\cdot7,\ u = 47^\circ\ 4'\ 45''\cdot9,\ e = 0\cdot37\,6847.$

These numbers show clearly that there is no orbit corresponding to the found values of ρ_1 and ρ_2.

When the agreement of the two values of $\log r$ is complete, we may find the remaining elements from the equations

$$\left\{\begin{aligned} \mathrm{Q}\sin i\sin\theta &= y_1z_2 - y_2z_1 \\ \mathrm{Q}\sin i\cos\theta &= x_1z_2 - x_2z_1 \\ \mathrm{Q}\cos i &= x_1y_2 - x_2y_1 \\ \mathrm{Q} &= r_1r_2\sin(v_2-v_1) > 0, \end{aligned}\right.$$

$$\tan\frac{v_1}{2} = \tan\left(45^\circ + \frac{\phi}{2}\right)\tan\frac{u_1}{2},\ \sin\phi = e,$$

$$\begin{aligned} r_1\sin(v_1+w) &= z_1\operatorname{cosec} i \\ r_1\cos(v_1+w) &= x_1\cos\theta + y_1\sin\theta, \end{aligned}$$

which need no further explanation.

Novotsherkassk:
1921 *October* 5.